Cournot Competition, Informational Feedback, and Real Efficiency*

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Abstract

We revisit the link between firm competition and real efficiency in a novel setting with informational feedback from financial markets. While intensified competition can decrease market power concentration in production, it reduces the value of proprietary information (on, e.g., market prospects) for speculators and discourages information production and price discovery in financial markets, with non-monotonic welfare effects. Market feedback can impact or even reverse the positive effects of competition on consumer welfare and real efficiency, especially when price becomes sufficiently informative for product decisions. The findings underscore the importance of considering the interaction between product market and financial market in antitrust policy, e.g., concerning the regulation of horizontal mergers. We demonstrate the robustness of the main results under dynamic trading, cross-asset trading and learning, etc.

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1 Introduction

The interaction and alignment between financial market efficiency and real efficiency constitute a long-standing topic in financial economics, as recently highlighted in studies on feedback effects (Goldstein et al., 2013; Goldstein and Yang, 2019; Goldstein, 2023). Unlike traditional theories on price formation (Grossman and Stiglitz, 1980; Hellwig, 1980; Glosten and Milgrom, 1985; Kyle, 1985), here the information flow is bi-directional: stock prices not only aggregate information from firms, but also contain new information effectively aggregated from traders, which real decision makers (e.g., managers) learn about and use to improve the efficacy of their decisions (e.g., investments and productions).

Against such a backdrop, we revisit the link between firm competition and real efficiency in the presence of stock market feedback. We show that the interaction between the financial market and the product market can undermine the positive effects of competition on real efficiency, contrary to conventional wisdom. Through a parsimonious model in which firm productions are endogenous to stock trading because of the informational feedback from stock prices, we provide new insights into competition and antitrust regulation.

Specifically, we consider a group of identical firms, each supervised by a manager, competing in a standard Cournot setting. The production decision of each firm depends on the assessment of uncertain market prospects, which managers can learn from stock prices. Meanwhile, stock prices aggregate the costly private information acquired by speculators who are incentivized by potential trading profits in financial markets. Firm managers then use the information extracted from stock prices to guide production decisions, which in turn affects firm valuation. The reliance of production decisions on stock prices establishes the feedback effect of the financial market on the real economy.

It is well known that firm competition increases total welfare by reducing market power concentration when firms engage in Cournot competition, which justifies the validity of antitrust regulations related to M&As, for example. However, when these firms are publicly traded, a countervailing force arises: intensified competition can reduce the information content of stock prices and decrease real efficiency. Therefore, intensified competition could generate a loss in total welfare rather than gains. Intuitively, with informational feedback, intensified competition generates both direct and indirect effects on total welfare. The direct effect entails the welfare gain as competition intensifies, reminiscent of that in conventional Cournot competition; the indirect effect comes from managerial learning from stock prices that aggregate individual speculators' information. Because intensified competition generally curbs the incentive for speculators to produce information, this translates into reduced information acquisition and incorporation into real decisions. A negative relationship between product market competition and total welfare ensues when the indirect effect is dominant.

The key mechanism behind the potential negative relationship between competition and welfare stems from feedback effects that influence the allocative efficiency of resources in production in uncertain environments. Managers set the capacity based on their estimation of future market prospects, relying on information learned from the stock market. In cases of managerial underestimation of market prospects, weaker competition enhances the informativeness of stock prices, correcting managers' downward biases, boosting production, and eventually improving resource allocation. Welfare increases if this production boost outweighs reduced total output caused by market power concentration. In contrast, when managers overestimate market prospects, reduced competition similarly improves information quality but corrects upward biases. This leads to reduced production and amplifies allocative efficiency losses, thus intensifying the negative welfare impact of market concentration.

Note that the negative link between competition and welfare depends on the relative gap in information production, rather than the absolute intensity, as competition intensifies. For example, when the information acquisition cost is high or low, information production either ceases or is in full scale, leading to a minimal change in information production when competition intensifies. Therefore, the market concentration channel dominates and thus competition always improves total welfare. In contrast, for an intermediate level of information cost, welfare-reducing competition always arises in the sense that any market structure with the total number of competing firms exceeding an exogenous threshold becomes suboptimal due to welfare loss related to deteriorated managerial learning alone.

We identify product profitability and market uncertainty as two key determinants of the relative strength of the aforementioned competing forces. Both factors can contribute to the direct effect of product market competition, although the positive effect of market uncertainty is more nuanced. With fixed information production for each stock, an increase in the number of stocks reduces the probability that all order flows are uninformative. However, intensified competition decreases information production, which indirectly leads to a large loss of welfare when amplified by the uncertainty of market prospects. Thus, one would expect the indirect effect to be dominant with low product profitability and high market uncertainty.

We extend the discussion in several important directions. First, we consider horizontal mergers by comparing the total welfare of a monopoly with that of a duopoly. Interestingly, a monopoly can dominate a duopoly in total welfare for an intermediate level of information production cost. When information production is too cheap or too costly, there is a small gap in the amount of information produced, and thus a monopoly is unlikely to be dominant.

Second, we consider cross-asset trading in which some traders with large investment opportunities (L-traders, including hedge funds, as introduced in Goldstein et al., 2014) can trade all stocks and the rest (S-traders such as individuals and some mutual funds) with small investment opportunities can only trade one stock. With cross-asset trading, the expected trading profits of L-traders, as competition intensifies, will first increase and then decrease, exhibiting an inverted U-shape pattern. Thus, the incentive for L-traders to acquire information will reach its maximum for a moderate level of competition. This differs sharply from S-traders, for whom the incentive of information production is always maximized in a monopoly. However, a negative relationship between competition and total welfare can still arise with L-traders, since the incentive of information production for L-traders will drop quickly after achieving its maximum level.

Third, we consider cross-asset learning in which market makers can observe the order flows of all stocks, rather than a single stock. This gives market makers more information advantages, reducing trading profits for both the S-traders and the L-traders. Actually, this makes S-traders more prone to competition compared to L-traders. Meanwhile, S-traders have a weaker incentive to acquire information compared to L-traders, implying that L-traders may "crowd out" S-traders due to cross-asset trading opportunities/abilities. Interestingly, we find that a negative relationship between product competition and total welfare can arise when S-traders are not fully crowded out by L-traders, which is more likely to occur if the cost of information production is relatively small.

Cochrane (2011) argues that discount rates mainly drive stock price movements instead of cash flows. We therefore also consider discount rates, and follow Dou et al. (2021) to assume that discount rates rise with competition. This further discourages speculators from acquiring information, exacerbating the negative effects of competition on information production and welfare.

Finally, we examine the impact of dynamic trading. Multiple trading rounds introduces market manipulation opportunities, especially on small firms (Edmans et al., 2015; Goldstein and Guembel, 2008; Banz, 1981; Acharya and Pedersen, 2005; Comerton-Forde and Putniņš, 2014). As competition reduces firm size, manipulation likelihood increases, further suppressing price informativeness and amplifying competition's negative welfare impact.

Our results have immediate implications for antitrust regulations in practice, where efficiency and welfare are the primary considerations. For example, regulators worry that M&A deals may substantially reduce competition and thus welfare costs by giving firms excessive market power to exploit other market participants and consumers (Guesnerie and Hart, 1985; Farrell and Shapiro, 1990; Landes and Posner, 1997). Horizontal mergers between direct competitors is particularly concerning. However, due consideration of the interaction between (financial) market efficiency and real efficiency is missing from existing antitrust rules.[1] The informational feedback from stock prices to real decisions generates a counter-intuitive implication: reduced competition can improve social welfare when the feedback effect from the financial market is sufficiently large. Using data from the U.S. market, we illustrate the importance of incorporating feedback effects in assessing the welfare impacts implications of mergers. Overall, these results highlight that feedback effects from the stock market are a critical factor in analyzing the welfare impact of horizontal mergers and the efficiency of market competition. To avoid misinterpreting merger and acquisition outcomes, antitrust regulatory bodies should take into account the interaction between the financial market and the real economy.

Literature. Our study adds to the literature on the feedback effects of financial markets on real efficiency. Early studies include Fishman and Hagerty (1989), Leland (1992), Dow and Gorton (1997), and Subrahmanyam and Titman (1999). As reviewed by Bond et al. (2012), and recently by Goldstein (2023), real decision makers (e.g., firm managers) can collect new information from stock prices to improve investments and production decisions (Foucault and Frésard, 2014; Edmans et al., 2015; Lin et al., 2019; Goldstein et al., 2013; Edmans et al., 2017; Goldstein and Yang, 2019). Central to this strand of literature is the alignment of market efficiency (i.e., the prediction power of stock prices for future cash flows) and real efficiency (i.e., the usefulness of stock prices for investment and production

[1]Section 7 of the Clayton Act, amended by the Celler-Kefauver Act later, prohibits mergers and acquisitions when the effect "may be substantially to lessen competition or to tend to create a monopoly." Consequently, the US Department of Justice (DOJ) and the Federal Trade Commission (FTC) have developed the Horizontal Merger Guidelines, delineating key factors and analytical frameworks, as well as many specific examples of how these principles can be applied in actual merger reviews. See, e.g., `https://www.justice.gov/atr/horizontal-merger-guidelines-0`.

decisions). These two notions of efficiency typically diverge under feedback effects (Dow and Gorton, 1997; Bond et al., 2012). Bai et al. (2016) derive a welfare-based measure of price informativeness and find a revelatory component has contributed significantly to the efficiency of capital allocation since 1960. Goldstein and Yang (2019) reveal a stark difference between market efficiency and real efficiency by considering multiple dimensions of information, generating interesting insights for optimal design of disclosure systems.[2]

Our paper differs by focusing on the welfare implications of intensified competition on real efficiency. In our model, product market competition can increase real efficiency by reducing firms' market power and decrease real efficiency by reducing information production by speculators. The two competing forces of reducing market power concentration and reducing information production jointly determine the impact of product market competition on social welfare.

A closely related study is Xiong and Yang (2021), which emphasizes the strategic information disclosure of firms. Our paper differs from theirs in the following three aspects, including: First, in their model, competition reduces firms' voluntary disclosure, ultimately leading to a decrease in economic efficiency. In contrast, we stress the role of information production by speculators and show that this mechanism alone can generate a negative relationship between competition and total welfare. Second, their analysis mainly compares a monopoly product market with a perfect competition market, whereas we consider any arbitrary number of firms and characterize general conditions under which competition decreases total welfare. Third, speculators no longer exogenously possess private information, but instead endogenously choose whether to become informed in our model.[3] Huang and Xu (2023) also explore the secondary market and product market competition, but focus on how initial stock holdings affect arbitrageurs' buying and thus entry decisions of potential uninformed entrants through feedback effects. More broadly, our paper relates to the aggregate implications of information production (e.g., Han and Yang, 2013). In particular, Angeletos et al. (2023) show that the two-way feedback between startup activity and investors beliefs can generate excessive and non-fundamental influences on firm activities and asset prices.

[2]More literature focusing on optimal disclosures include: Chen et al. (2021); Edmans et al. (2015); Boleslavsky et al. (2017); Gao and Liang (2013) and Jayaraman and Wu (2019).

[3]More precisely, Xiong and Yang (2021) also consider endogenous information acquisition by speculators in their Section 5.3. A key difference is that when the number of firms increases, information acquisition decreases in the extensive margin in our paper, while Xiong and Yang (2021) document a different pattern in which the extensive margin of information acquisition increases while the intensive margin decreases. This further suggests that this insight is robust to different ways of modeling information acquisition.

Our study is also related to the long-standing literature investigating the relationship between competition and economic efficiency and its implications for antitrust regulations. Dating back to Smith (1776) and Cournot (1838), the traditional wisdom — the existence of market power can generate market inefficiencies and reduce welfare by raising price and suppressing output — has greatly influenced the evolution of the Horizontal Merger Guidelines (Nocke and Whinston, 2022).[4] On the one hand, the unilateral effect analysis emphasizes the trade-off between post-merger market power and potential synergies (see, e.g., Williamson, 1968; Farrell and Shapiro, 1990; Nocke and Whinston, 2022).[5] On the other hand, the coordinated effect analysis concerns implicit anti-competitive coordination from mergers in the absence of explicit communication (see, e.g., Compte et al., 2002; Miller and Weinberg, 2017; Porter, 2020). Röller et al. (2001) and Asker and Nocke (2021) offer comprehensive surveys of this vast literature before 2001 and more recent developments, respectively. In addition, Peress (2010) analyzes how product market competition influences stock price informativeness, which in turn affects capital allocation.

We examine not only the potential negative impact of firm competition on price informativeness but also the informational feedback from stock prices to production decisions, with novel welfare and policy implications. In particular, we show that without cost synergies that are commonly assumed in prior studies, informational feedback from stock market alone can affect and even reverse the welfare effects of a horizontal merger. Thus, our analysis reveals the feedback effect to be an important and indispensable factor in analyzing the welfare impact of horizontal mergers and the efficiency of market competition.

Finally, several recent studies explore direct evidence for merger-specific efficiency (Ashenfelter et al., 2015; Braguinsky et al., 2015), and characterize what counts as an efficiency (Hemphill and Rose, 2017; Geurts and Van Biesebroeck, 2019). Covarrubias et al. (2020) identify good and bad concentrations at the aggregate and industry level in the United States over the past three decades. Our paper contributes to the discussion of positive merger-specific efficiencies by exploring a new channel through feedback effects between the product market and the financial market. Two other related papers, Edmans et al. (2012) and Luo (2005), similarly explore the feedback effect in mergers and acquisitions. Both em-

[4] The Horizontal Merger Guidelines feature two key considerations: unilateral price effects and coordinated effects. Other concerns include pro-competitive forces such as market entry and dynamic considerations (see, e.g., Mermelstein et al., 2020; Nocke and Whinston, 2010).

[5] Recently, a growing literature evaluates "merger simulations" to quantify unilateral price effects and welfare impacts (Werden and Froeb, 1994; Weinberg, 2011; Björnerstedt and Verboven, 2016; Nevo, 2000).

phasize how learning by insiders from outsiders' information affects the decision for M&As but do not focus on the link between competition and efficiency as we do.

The remainder of the paper is organized as follows: Section 2 sets up the model. Section 3 characterizes the equilibrium. Section 4 revisits the relationship between production competition and real efficiency in the presence of feedback effects. Section 5 extends the baseline model and discusses the robustness of the main results. Finally, Section 6 concludes. All proofs are relegated to the appendix.

2 Model Setup

We embed feedback from stock prices to product decisions under market competition into an otherwise standard Cournot model. Consider $n \geq 2$ identical firms competing in production quantity, and each firm's equity is traded on a public stock exchange. Time is discrete and indexed by $t \in \{0, 1\}$. At $t = 0$, a group of speculators decide whether to acquire private information on the market prospects of the product and subsequently decide how to trade stocks.[6] Then, the manager of each firm makes a production decision, taking into account the production strategies of other firms and the trading on the stock exchange at $t = 0$. Finally, at $t = 1$, the cash flows for all firms are realized. The key departure from the Cournot model is that managers in our setting can learn and use information contained in stock prices for their production decisions.

The product market. Let q_i denote the output level of the ith firm, where $i \in \{1, \ldots, n\}$.[7] Denote the total supply of the product by $Q = \sum_{i=1}^{n} q_i = q_i + q_{-i}$, where $-i$ denotes all other firms. As in Xiong and Yang (2021), the market clearing price P is given by: $P = A - bQ$. Here, $b > 0$ indicates the sensitivity of demand to price and $A > 0$ captures the possible market prospect of the product. Depending on a relevant economic state $\omega \in \{H, L\}$, the

[6] We follow the literature by assuming that speculators only acquire information once (See, e.g., Gao and Liang, 2013; Goldstein et al., 2014; Dow et al., 2017; Xiong and Yang, 2021). The effects of introducing multiple rounds of trading will be discussed in Section 5.

[7] We focus on Cournot competition (i.e., quantity competition), rather than Bertrand price competition, for the following two reasons. First, in canonical Bertrand competition, the total welfare is independent of the total number of competing firms. Second, as shown in Kreps and Scheinkman (1983), the quantity (capacity) pre-commitment and the Bertrand price competition yield Cournot outcomes. In addition, we anticipate that Bertrand competition can weaken our result even with differentiated products. For example, Vives (1985) shows that prices and profits are generally higher and quantities are lower in Cournot competition than in Bertrand competition. Therefore, Bertrand competition can enhance the effect of market concentration, potentially reducing the relative significance of information feedback.

realization of the market prospect is given by $A(\omega) = A_\omega$, where $A_H > A_L > 0$. Both states are equally likely ex ante, i.e., $\Pr(\omega = H) = \Pr(\omega = L) = 1/2$. Given the production decisions $\{q_i\}_{1\leq i\leq n}$, the ith firm receives an operating profit given by:

$$TP_i(q_i) = q_i(A - bQ - MC), \tag{1}$$

where MC is a constant marginal production cost. Without loss of generality, we assume that $A_H > A_L \geq MC$. To highlight the core mechanism, we leave out financing constraints.

All firms decide simultaneously on the production level q_i at time $t = 0$. Each firm manager maximizes the expected value of the firm after the stock prices are observed. In other words, conditional on the information observed, $\mathcal{F}_m$, at $t = 0$, the firm manager chooses the output level q_i to maximize:

$$V_i(q_i) = \mathbb{E}[TP_i(q_i) \mid \mathcal{F}_m]. \tag{2}$$

The stock market. All firms are publicly traded by three types of investors: (i) a continuum of risk-neutral speculators who can choose to acquire costly information; (ii) a group of liquidity traders for each firm $i \in \{1, \cdots, n\}$, who jointly submit an aggregate order $z_i \sim U([-1, 1])$, independently and uniformly distributed over $[-1, 1]$ across the identity of the firm i; and (iii) a set of risk neutral market makers. The free entry of market makers implies that each makes zero profit in equilibrium.

For each firm i, let $\alpha_i \in [0, 1]$ denote the size of speculators acquiring costly information at $t = 0$ as in Foucault and Frésard (2014). To endogenously determine the amount α_i of informed speculators, we assume that each speculator k must pay a cost $c > 0$ to become informed, i.e., receiving an informative signal $m_k^i \in \{H, L\}$.[8] With precision $\theta > \frac{1}{2}$, the signal structure is given by:

$$\Pr\left(m_k^i = H|\omega = H\right) = \Pr\left(m_k^i = L|\omega = L\right) = \theta. \tag{3}$$

Conditional on the realization of ω, m_k^i is independently and identically distributed across speculators (as in Goldstein et al., 2013; Dow et al., 2017). Upon observing the signal m_k^i, the kth informed speculator can choose to trade x_k^i shares of the ith firm, where $x_k^i \in [-1, 1]$ as in Dow et al. (2017). Thus, the aggregate demand for the ith stock from speculators is

[8] The superscript "i" in m_k^i is used to indicate that the kth speculator is trading the ith stock.

given by: $x_i = \int_0^{\alpha_i} x_k^i dk$. Recall that all liquidity traders submit an aggregate order z_i that is uniformly distributed. The total order flow f_i for the ith stock is: $f_i = z_i + x_i$.

As in Kyle (1985), the order flow f_i in each stock i is absorbed by market makers, and the stock price s_i reflects the expected value of the firm conditional on the total order flow:

$$s_i(f_i) = \mathbb{E}[V_i \mid f_i]. \tag{4}$$

Equilibrium definition. The equilibrium concept that we use is perfect Bayesian equilibrium, which consists of: (i) a production strategy for each manager that maximizes the expected firm value given the information conveyed in stock prices; (ii) an information production strategy and a trading strategy for speculators that maximize the expected trading profit given all others' strategies; (iii) a price-setting strategy for market makers that allows them to break even in expectation given all others' strategies; (iv) managers and market makers update their beliefs about the economic state according to the Bayes rule; and (v) each player's belief about other players' strategies is correct in equilibrium.

3 Equilibrium Characterization

We solve the model backward. We first derive the equilibrium strategy of firms, taking as a given the amount α_i of informed speculators for each firm i, and then we endogenize α_i. As shown later, an informed speculator k with a private signal m_k^i always buys one share of the stock of the ith firm when $m_k^i = H$, and sells one share when $m_k^i = L$. Given this observation, we can now investigate the production strategies of firms and the pricing rules for stocks in equilibrium.

Let us first consider the limit where the information acquisition cost c is sufficiently high that all speculators abstain from acquiring information. When this occurs, the stock price is uninformative and the market outcome reduces to the standard Cournot competition outcome with n identical firms. Therefore, each firm produces an identical output:

$$q_M = \frac{\bar{A} - MC}{(n+1)b}, \tag{5}$$

where $\bar{A} = \frac{1}{2}(A_H + A_L)$.

This can be compared with the market outcome when the actual market prospect $A(\omega)$

is publicly known to all market participants. Specifically, when $A(\omega) = A_H$, each firm produces a quantity of $q_H = \frac{A_H - MC}{(n+1)b}$, making a profit of $s_H = \frac{(A_H - MC)^2}{(n+1)^2 b}$. Similarly, when $A(\omega) = A_L$, each firm produces $q_L = \frac{A_L - MC}{(n+1)b}$, making a profit of $s_L = \frac{(A_L - MC)^2}{(n+1)^2 b}$. In contrast, in the absence of information produced by speculators, the equilibrium output q_M under uncertainty is just the expectation of outputs in both states, i.e., $q_M = \frac{1}{2}(q_H + q_L)$.

Next, we consider the case of informative stock trading. Intuitively, due to information-based speculative trading, stock prices contain useful information for managers to guide production decisions. Thus, to solve for the production strategy with informational feedback effects, we need to analyze stock pricing rules in equilibrium. Following Kyle (1985), market makers set stock prices based on the updated belief about the value of firms, given the total order flow observed. Given the information structure in Equation (3), by the law of large numbers (Dow et al., 2017), the aggregate order of informed speculators is $x_i = \alpha_i(2\theta - 1)$ when $\omega = H$, generating a total order flow of $f_i = \alpha_i(2\theta - 1) + z_i$. Similarly, if $\omega = L$, then: $f_i = -\alpha_i(2\theta - 1) + z_i$.

In summary, market makers condition the pricing on the observed total order flow, which aggregates the information from the trading activities of informed speculators. Therefore, the stock price contains valuable information for managers, which establishes an information feedback channel to the real economy. As shown in Lemma 1, the optimal production strategies of firms explicitly depend on stock prices.

Lemma 1. *Given the measures of informed speculators $\{\alpha_i\}_{1 \le i \le n}$, the equilibrium stock price for the ith firm is given by:*

$$s_i(f_i) = \begin{cases} s_H, & \text{if } f_i > \gamma_i \\ s_M^i, & \text{if } -\gamma_i \le f_i \le \gamma_i \\ s_L, & \text{if } f_i < -\gamma_i \end{cases}, \tag{6}$$

where $s_H = \frac{(A_H - MC)^2}{(n+1)^2 b}$, $s_M^i = \frac{1}{4(n+1)^2 b}\left\{2\left((A_H - MC)^2 + (A_L - MC)^2\right) - \beta_i (A_H - A_L)^2\right\}$, $s_L = \frac{(A_L - MC)^2}{(n+1)^2 b}$, $\gamma_i = 1 - \alpha_i(2\theta - 1)$, and $\beta_i = \prod_{j \ne i} \gamma_j$.

Furthermore, given all stock prices $\{s_i\}_{1 \le i \le n}$, the ith firm produces an output of:

$$q_i^* = \begin{cases} q_H, & \text{if } s_j = s_H \text{ for some } j \\ q_M, & \text{if } s_j = s_M^j \text{ for all } j \\ q_L, & \text{if } s_j = s_L \text{ for some } j \end{cases}, \tag{7}$$

where $q_H = \frac{A_H - MC}{(n+1)b}$, $q_L = \frac{A_L - MC}{(n+1)b}$, *and* q_M *is given by Equation (5).*

We make three comments on Lemma 1. First, the three conditions in Equation (6), as well as those in Equation (7), are mutually exclusive, which rules out the possibility of observing both $s_i = s_H$ and $s_j = s_L$ for some $i \neq j$.[9] Thus, the optimal production strategy q_i^* is well defined. Second, we can directly verify that $s_H > s_M^i > s_L$, which implies that the equilibrium stock price s_i increases weakly in the total order flow f_i. This result is consistent with those of the existing literature on feedback effects (Foucault and Frésard, 2014; Dow et al., 2017; Lin et al., 2019). Third, managers choose equilibrium output levels based on observed stock prices. Obviously, $q_H > q_M > q_L$, which implies that q_i^* generally tends to increase with stock prices.

We now proceed to analyze the optimal behavior of speculators in equilibrium. Specifically, we first derive the optimal trading strategy of an informed speculator and then calculate the resulting expected trading profits, which are summarized in Lemma 2 below.

Lemma 2. *For speculators that focus on the* i*th stock, the optimal trading strategy is to long one share (that is,* $x_k^i = +1$*) when* $m_k^i = H$ *and short one share (that is,* $x_k^i = -1$*) when* $m_k^i = L$*. The resulting expected trading profit is:*

$$\Pi_i(\boldsymbol{\alpha}) = \frac{\gamma_i(2\theta - 1)\left(2 + (n-1)\beta_i\right)}{2(n+1)^2 b}\left(\bar{A} - MC\right)\left(A_H - A_L\right).$$

Lemma 2 verifies the intuition that an informed speculator always follows his own signal, i.e., he longs the stock after receiving good news and shorts it after bad news. Also note that $\Pi_i(\boldsymbol{\alpha})$ depends on all $\{\alpha_i\}_{1 \leq i \leq n}$ through γ_i and β_i. Furthermore, the expected trading profit $\Pi_i(\boldsymbol{\alpha})$ strictly increases both in the average profitability, as measured by $\left(\bar{A} - MC\right)$, and in the uncertainty about the market prospects, as measured by $(A_H - A_L)$.

Finally, Lemma 2 is an important intermediate step in understanding the incentive for information production. Specifically, when acquiring costly information on market prospects, an uninformed speculator balances between the cost of information production $c > 0$ and the value of proprietary information $\Pi_i(\boldsymbol{\alpha})$. Since all firms are identical in the Cournot competition, we hereafter focus on the symmetric case $\alpha_i = \alpha$ ($\forall\ 1 \leq i \leq n$) and define:

$$\Pi(\alpha) := \Pi_i(\boldsymbol{\alpha}) = \frac{\gamma(2\theta - 1)\left(2 + (n-1)\gamma^{n-1}\right)}{2(n+1)^2 b}\left(\bar{A} - MC\right)\left(A_H - A_L\right), \tag{8}$$

[9] To see this, given that $s_i = s_H$, the state consistent with the order flow of noise trading can only admit $\omega = H$, contradicting $s_j = s_L$ which fully reveals that $\omega = L$.

where $\gamma = 1 - \alpha(2\theta - 1)$.

Note that $\Pi(\alpha)$ in Equation (8) strictly decreases in α, i.e., $\frac{\partial \Pi(\alpha)}{\partial \alpha} < 0$. Thus, the value of private information decreases when more agents choose to do so, implying that information acquisition is a strategic substitute among speculators.

Intuitively, when the cost of information acquisition is large enough such that $\Pi(0) \leq c$, no speculator has an incentive to acquire education. However, when the cost parameter is sufficiently small such that $c \leq \Pi(1)$, all speculators choose to acquire information. Together, these two conditions establish two cut-off points, including an upper bound $\overline{c} = \Pi(0)$ and a lower bound $\underline{c} = \Pi(1)$. Specifically, we define:

$$\overline{c}_n = \frac{(2\theta - 1)}{2(n+1)b} \left(\bar{A} - MC\right)(A_H - A_L) \tag{9}$$

and

$$\underline{c}_n = \frac{(2\theta - 1)(1-\theta)\left(2 + (n-1)(2-2\theta)^{n-1}\right)}{(n+1)^2 b} \left(\bar{A} - MC\right)(A_H - A_L) \tag{10}$$

Let $\widehat{\alpha}$ denote the optimal intensity of information acquisition.

Proposition 1 (Information Acquisition Intensity)**.**

(i) When $c \geq \overline{c}_n$, there is a unique symmetric equilibrium with no information production ($\widehat{\alpha} = 0$);

(ii) When $0 \leq c \leq \underline{c}_n$, then $\widehat{\alpha} = 1$ in the unique equilibrium; and

(iii) When $\underline{c}_n < c < \overline{c}_n$, there is a unique interior equilibrium with $\widehat{\alpha} \in (0,1)$ such that $\Pi(\widehat{\alpha}) = c$.

Two comments are in order. When $\Pi'(\widehat{\alpha}) < 0$, an interior solution $\widehat{\alpha}$ is said to be locally stable because when we start with $\alpha < \widehat{\alpha}$, more speculators find it optimal to acquire information, increasing the intensity of information acquisition and vice versa. Moreover, the incentive to acquire and trade on private information is negatively associated with the cost of information production. Such an equilibrium on information acquisition is reminiscent of that in Grossman and Stiglitz (1980). A sufficiently large cost preempts the incentive to acquire information, and thus the informational feedback effect disappears. In general, the information content of stock prices depends on the amount of informed speculators in the stock market, which is pinned down uniquely by the information cost and other model parameters.

4 Competition and Efficiency Under Feedback Effects

We now establish that product market competition can decrease the incentive for speculators to produce information and then analyze the efficiency implications of firm competition with informational feedback from stock prices. Interestingly, reduced competition in the stock market can enhance informational efficiency, leading to allocative efficiency gains that significantly alter the efficiency implications of product market competition. When the feedback effect is sufficiently strong, Cournot competition may even produce negative welfare effects.

4.1 Information Production

We first analyze how information production, measured by the equilibrium size of informed speculators $\widehat{\alpha}_n := \widehat{\alpha}(n)$, varies with the number of firms n in the product market. For simplicity, we focus on the interior solution case; otherwise, we expect that $\partial\widehat{\alpha}_n/\partial n = 0$ under corner solutions. Then, we rewrite the equilibrium condition as:

$$\Pi(\widehat{\alpha}) = \Pi(n, \widehat{\alpha}_n) = c \tag{11}$$

A direct application of the implicit function theorem implies the following:

Proposition 2 (Competition and Information Production)**.** *When an interior solution $\widehat{\alpha}_n \in (0,1)$ exists $c \in (\underline{c}, \overline{c})$, $\widehat{\alpha}_n$ strictly decreases in n, that is, $\frac{\partial\widehat{\alpha}_n}{\partial n} < 0$.*

Proposition 2 verifies that the amount $\widehat{\alpha}_n$ of informed speculators increases as competition weakens driven by stronger incentives to acquire information. This result is consistent with empirical evidence in Farboodi et al. (2022) in which investors have relatively more data on large firms than on small ones because the incentive for speculators to produce information increases with reduced competition, which raises both firm profitability and size.

Furthermore, it is also worth examining how information production is affected by changes in other model parameters related to the product market, including the unit production cost MC, the price sensitivity of demand b and market prospect parameters A_H and A_L. Again, we can apply the implicit function theorem to the equilibrium condition (11) to derive:

Corollary 1. *When $c \in (\underline{c}_n, \overline{c}_n)$ so that an interior solution $\widehat{\alpha}_n \in (0,1)$ exists, the equilibrium features $\frac{\partial\widehat{\alpha}_n}{\partial MC} < 0$, $\frac{\partial\widehat{\alpha}_n}{\partial b} < 0$, $\frac{\partial\widehat{\alpha}_n}{\partial A_H} > 0$, and $\frac{\partial\widehat{\alpha}_n}{\partial A_L} < 0$.*

Information production, measured by the amount $\widehat{\alpha}_n$ of informed speculators, decreases with the production cost MC. This result can be understood by analyzing the expected trading profit $\Pi(\alpha)$, which is lower for a higher MC. Obviously, a lower expected trading profit will reduce the incentive for speculators to produce information, decreasing the equilibrium amount of information production. Similarly, when demand becomes relatively more sensitive to price (i.e., $b\uparrow$), the amount $\widehat{\alpha}_n$ of informed speculators will also decrease, since the expected trading profit Π is lower for a higher b. Furthermore, $\widehat{\alpha}_n$ increases in A_H and decreases in A_L. To understand these, note that the expected trading profit Π increases in the market uncertainty that is proportional to $(A_H - A_L)^2$. Therefore, a larger gap of $(A_H - A_L)$ increases the expected trading profit of informed speculators, inducing them to acquire more information.

4.2 Feedback Effects and Allocative Efficiency

The previous section shows that reduced competition in the product market enhances the information efficiency of the stock market. We now examine how this improvement in price informativeness affects allocative efficiency in the real economy. The central idea is that, through the feedback effect, managers' ability to learn from stock prices helps correct potential underestimation or overestimation of the market prospect $A(\omega)$, improving their production decisions and thereby increasing real efficiency via more effective information production.

We begin by introducing the probability of misallocation, which stems from managerial underestimation or overestimation of the market prospect. From Lemma 1,

$$\Pr(\forall i: q_i^* = q_M \mid \omega = H) = (\widehat{\gamma}_n)^n \quad \text{and} \quad \Pr(\forall i: q_i^* = q_H \mid \omega = H) = 1 - (\widehat{\gamma}_n)^n$$

Thus, with probability $1 - (\widehat{\gamma}_n)^n$, the true state $\{\omega = H\}$ is revealed through stock prices, allowing managers to correctly estimate the market prospect A_H. As a result, both the aggregate output and the price to align with those in Cournot competition under complete information; that is, $Q_H(n) = \frac{n(A_H - MC)}{b(n+1)}$ and $P_H(n) = \frac{A_H + nMC}{(n+1)}$. However, with complementary probability $(\widehat{\gamma}_n)^n$, stock prices remain uninformative, leading managers to underestimate the market prospect. This results in an inefficiently lower output $Q_M(n) = \frac{n(\bar{A} - MC)}{b(n+1)} < Q_H$ and a higher price $P_{MH}(n) = P_H(n) + \frac{n(A_H - A_L)}{2(n+1)} > P_H(n)$. Thus, $(\widehat{\gamma}_n)^n$ represents the probability of misallocation when the true state is $\omega = H$. Similarly, misallocation occurs with probability $(\widehat{\gamma}_n)^n$ when the true state is $\omega = L$, where managers may overestimate the market prospect.

Next, we measure total welfare, $W(n;\omega)$, which includes both firm profits, $\Gamma_\omega(n) = \mathbb{E}\left[\sum_{i=1}^n TP_i \mid \omega\right]$ and consumer surplus, $CS_\omega(n) = \frac{1}{2}(A(\omega) - P)Q$. Formally, total welfare is given by:

$$W(n;\omega) = \frac{1}{2}(A(\omega) - P)Q + \mathbb{E}\left[\sum_{i=1}^{n} TP_i \mid \omega\right], \tag{12}$$

Since $A(\omega)$ is random, the expected total welfare and consumer welfare are given by $\overline{W} = \mathbb{E}_\omega[W(n;\omega)]$ and $\overline{CS} = \mathbb{E}_\omega[CS_\omega(n)]$, respectively.

Allocative efficiency gains. We now analyze how gains (or losses) in allocative efficiency arise through feedback effects. Figure 1 illustrates the source of these efficiency changes by comparing the total welfare between n firms and $(n-1)$ firms when the true state is $\omega = H$. Specifically, in the case of n firms, with probability $1-(\widehat{\gamma}_n)^n$, managers correctly estimate the market prospect A_H, resulting in an output of $Q_H(n)$ and corresponding welfare represented by the area Area$(ABNM)$. Conversely, with complementary probability $(\widehat{\gamma}_n)^n$, the output $Q_M(n)$ is lower due to managerial underestimation of the market prospect, and the welfare is represented by the area Area$(ABFE)$. By weighting these two areas by the probabilities of $(\widehat{\gamma}_n)^n$ and $1-(\widehat{\gamma}_n)^n$, we obtain the expected total welfare $W_H(n)$ given $\omega = H$, which corresponds to the blue trapezoid area, Area$(ABHG)$.

In contrast, when there are $(n-1)$ firms, with probability $1-(\widehat{\gamma}_{n-1})^{n-1}$, the output is $Q_H(n-1)$, and the corresponding welfare is represented by the area Area$(ABLK)$; with complementary probability $(\widehat{\gamma}_{n-1})^{n-1}$, managers underestimate the market prospect and the output is $Q_M(n-1)$, resulting in a lower welfare represented by the area Area$(ABDC)$. By weighting these two areas by the probabilities $(\widehat{\gamma}_{n-1})^{n-1}$ and $1-(\widehat{\gamma}_{n-1})^{n-1}$, we obtain the expected total welfare $W_H(n-1)$ given $\omega = H$, which corresponds to the blue trapezoid area Area$(ABJI)$.

The welfare gain due to reduced competition is then given by $W_H(n-1;\omega) - W_H(n;\omega)$, which is positive only when Area$(ABJI) >$ Area$(ABHG)$ holds. Indeed, this condition holds when the price impact from reduced competition is negative. To assess the price impact, note that $\overline{P}_H(n) = (\widehat{\gamma}_n)^n \times P_{HM} + (1-(\widehat{\gamma}_n)^n) \times P_H$ and $\overline{P}_H(n-1) = (\widehat{\gamma}_{n-1})^{n-1} \times P_{HM}(n-1) + (1-(\widehat{\gamma}_{n-1})^{n-1}) \times P_H(n-1)$. Thus, the price effect from reduced competition

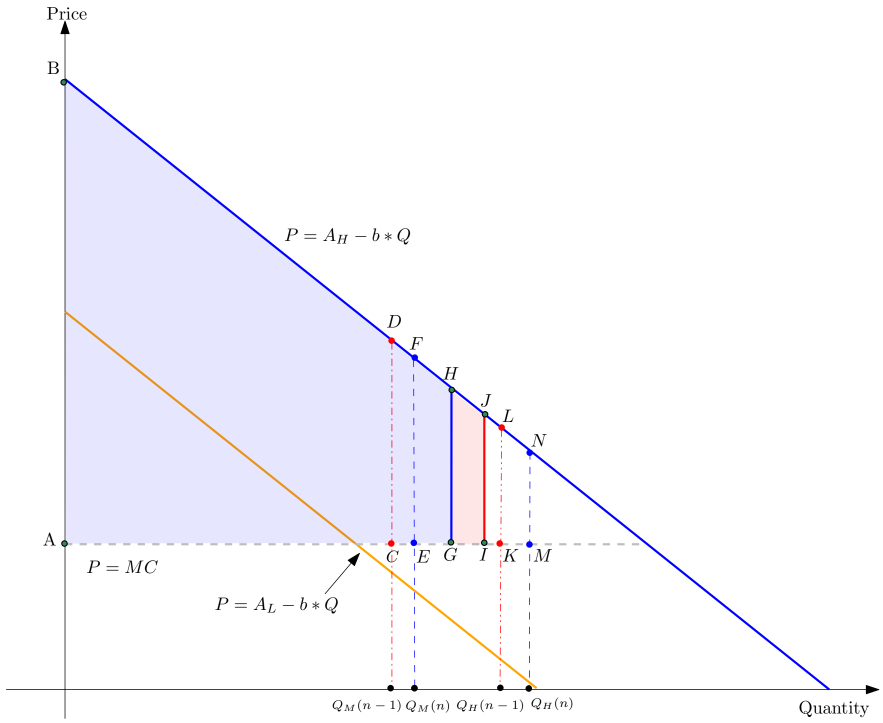

Figure 1: Allocative Efficiency Gain ($\omega = H$)

Notes: When there are n firms, with probability $1-(\widehat{\gamma}_n)^n$, the output is $Q_H(n)$ and the corresponding welfare is $Area(ABNM)$; with complementary probability $(\widehat{\gamma}_n)^n$, the output is $Q_M(n)$ and the welfare is $Area(ABFE)$. The expected welfare $W_H(n)$ then is the average of $Area(ABFE)$ and $Area(ABNM)$ weighted by $(\widehat{\gamma}_n)^n$ and $1-(\widehat{\gamma}_n)^n$, respectively. Similar discussion applies when there are $(n-1)$ firms, and the expected welfare $W_H(n-1)$ is the average of $Area(ABLK)$ and $Area(ABDC)$ weighted by $(\widehat{\gamma}_{n-1})^{n-1}$ and $1-(\widehat{\gamma}_{n-1})^{n-1}$, respectively. If $(\widehat{\gamma}_{n-1})^{n-1}$ is sufficiently small (compared with $(\widehat{\gamma}_n)^n$) such that condition (13) holds, an allocative efficiency gain will arise, i.e.,$W_H(n) = Area(ABHG) < W_H(n-1) = Area(ABJI)$.

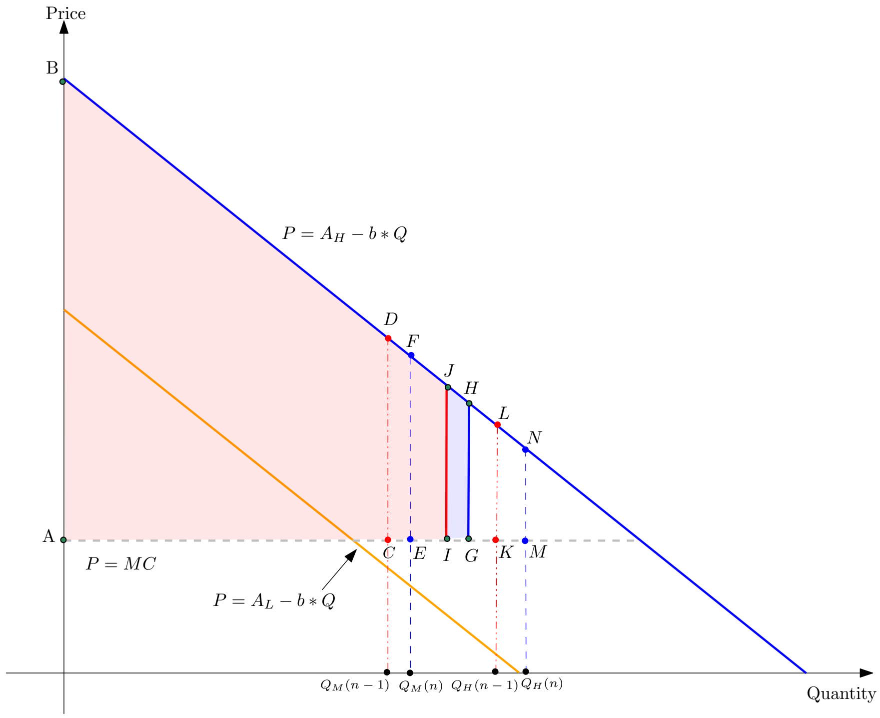

Figure 2: Allocative Efficiency Loss ($\omega = H$)

Notes: When there are n firms, with probability $1-(\widehat{\gamma}_n)^n$, the output is $Q_H(n)$ and the corresponding welfare is $Area(ABNM)$; with complementary probability $(\widehat{\gamma}_n)^n$, the output is $Q_M(n)$ and the welfare is $Area(ABFE)$. The expected welfare $W_H(n)$ then is the average of $Area(ABFE)$ and $Area(ABNM)$ weighted by $(\widehat{\gamma}_n)^n$ and $1-(\widehat{\gamma}_n)^n$, respectively. Similar discussion applies when there are $(n-1)$ firms, and the expected welfare $W_H(n-1)$ is the average of $Area(ABLK)$ and $Area(ABDC)$ weighted by $(\widehat{\gamma}_{n-1})^{n-1}$ and $1-(\widehat{\gamma}_{n-1})^{n-1}$, respectively. If $(\widehat{\gamma}_{n-1})^{n-1}$ is not sufficiently small (compared with $(\widehat{\gamma}_n)^n$) such that condition (13) does not hold, an allocative efficiency loss will arise, i.e., $W_H(n) = Area(ABHG) > W_H(n-1) = Area(ABJI)$.

in the state $\omega = H$ is:

$$\Delta\overline{P}_H(n) = \overline{P}_H(n-1) - \overline{P}_H(n) = \frac{A_H - MC}{n(n+1)} + \frac{A_H - A_L}{2n(n+1)}\left[\left(n^2-1\right)\left(\widehat{\gamma}_{n-1}\right)^{n-1} - n^2\left(\widehat{\gamma}_n\right)^n\right]$$

Interestingly, reduced competition can lead to a negative price impact (i.e., $\Delta\overline{P}_H(n) < 0$) when the reduction in misallocation probability is sufficiently significant, such that:

$$\left(\widehat{\gamma}_{n-1}\right)^{n-1} < \frac{n^2}{(n^2-1)}\left(\widehat{\gamma}_n\right)^n - \frac{2\left(A_H - MC\right)}{(n^2-1)\left(A_H - A_L\right)}. \tag{13}$$

Intuitively, this inequality holds if reduced competition significantly improves information production and lowers the value of $(\widehat{\gamma}_{n-1})^{n-1}$. This enables managers to better correct their underestimation of the market prospect A_H and reduce misallocation. This scenario corresponds to the allocative efficiency gain depicted in Figure 1. In contrast, Figure 2 illustrates allocative efficiency losses under weak feedback effects when equation (13) is violated.

Two clarifications are necessary regarding allocative efficiency gains (or losses). First, allocative efficiency gains cannot occur in the state $\omega = L$, as managers overestimate the market prospect A_L. Reduced competition ($n \downarrow$) decreases both $Q_L(n) = \frac{n(A_L - MC)}{b(n+1)}$ (when the state is revealed) and $Q_M(n) = \frac{n(\bar{A} - MC)}{b(n+1)}$ (when prices are uninformative). Furthermore, improved price informativeness under reduced competition corrects managers' upward biases, causing them to further reduce output and thus increase prices. Hence, reduced competition always results in higher prices in the low state. Second, the high state ($\omega = H$) has a greater impact on total welfare due to its larger market size. Since allocative efficiency gains from feedback effects arise mainly in the high state, these gains dominate welfare outcomes only when market uncertainty is sufficiently large, making welfare in the low state relatively less important.

4.3 Competition and Real Efficiency

We now formally analyze the efficiency implications of product market competition with feedback effects. Traditional wisdom claims that standard Cournot competition always improves economic efficiency and that imperfect/insufficient competition, such as oligopolies and monopolies, often leads to dead weight loss (Willner, 1989). However, existing studies on Cournot competition ignore the feedback effects of the financial market. Proposition 2 explains why the traditional argument may fail: product market competition lowers speculators' incentives to acquire information, leading to inefficient production decisions. The previous section also shows how feedback effects can create allocative efficiency gains, potentially reversing the link between product competition and welfare.

Specifically, the expected total welfare in the presence of feedback effects is given by:

$$\overline{W}\left(\widehat{\alpha}_n, n\right) = \frac{n(n+2)}{8b(n+1)^2}\left(4\left(\bar{A} - MC\right)^2 + \left(1 - \widehat{\gamma}_n{}^n\right)\left(A_H - A_L\right)^2\right), \tag{14}$$

where $\widehat{\gamma}_n = 1 - \widehat{\alpha}_n(2\theta - 1)$. Correspondingly, consumer welfare is given by:

$$\overline{CS}\left(\widehat{\alpha}_n, n\right) = \frac{n^2}{8b(n+1)^2}\left(4\left(\bar{A} - MC\right)^2 + \left(1 - \widehat{\gamma}_n{}^n\right)\left(A_H - A_L\right)^2\right). \tag{15}$$

Note that both $\overline{W}\left(\widehat{\alpha}_n, n\right)$ and $\overline{CS}\left(\widehat{\alpha}_n, n\right)$ strictly increase with average profitability $(\overline{A} - MC)$ and market uncertainty $(A_H - A_L)$. Notably, $\overline{W}\left(\widehat{\alpha}_n, n\right)$ becomes more sensitive to $(A_H - A_L)$ as the number of informed speculators increases (i.e., $\widehat{\alpha}_n \uparrow$), reducing the probability of misallocation $(\widehat{\gamma}_n)^n$. This effect arises only due to informational feedback.

Next, we examine the relationship between total welfare and firm competition in the presence of feedback effects and investigate whether total welfare $\overline{W}(\widehat{\alpha}_n, n)$ can be negatively associated with the competition parameter n. To this end, we compute the total derivative of total welfare $\overline{W}(\widehat{\alpha}_n, n)$ with respect to n, the number of firms, as follows:

$$\frac{d\overline{W}(\widehat{\alpha}_n, n)}{dn} = \underbrace{\frac{\partial \overline{W}(\widehat{\alpha}_n, n)}{\partial n}}_{\text{Competition Effects}} + \underbrace{\frac{\partial \overline{W}(\widehat{\alpha}_n, n)}{\partial \widehat{\alpha}_n}\frac{\partial \widehat{\alpha}_n}{\partial n}}_{\text{Feedback Effects}}. \tag{16}$$

Equation (16) decomposes the total welfare effect into direct competition effects and feedback effects. Obviously, one can verify that $\frac{\partial \overline{W}(\widehat{\alpha}_n,n)}{\partial n} > 0$, which is consistent with the conventional wisdom that product market competition tends to increase total welfare (see, e.g., Willner, 1989). Meanwhile, since Proposition 2 establishes that $\frac{\partial \widehat{\alpha}_n}{\partial n} < 0$ (i.e., fierce product competition discourages information production), it might be possible for $\frac{d\overline{W}(\widehat{\alpha}_n,n)}{dn}$ to be negative when $\frac{\partial \overline{W}(\widehat{\alpha}_n,n)}{\partial \widehat{\alpha}_n}$ is positive and sufficiently large. Note that $\frac{\partial \overline{W}(\widehat{\alpha}_n,n)}{\partial \widehat{\alpha}_n}$ measures the sensitivity of total welfare to the amount of information produced by speculators $\widehat{\alpha}_n$ in the stock market. Intuitively, as $\widehat{\alpha}_n$ increases, a higher level of informativeness of the stock market improves real efficiency in production, and thus a positive value of $\frac{\partial \overline{W}(\widehat{\alpha}_n,n)}{\partial \widehat{\alpha}_n}$ follows.[10]

Lemma 3 (Competition and Real Efficiency)**.**

Define $G_1(A_H, A_L, MC) = 2 + 8\left(\bar{A} - MC\right)^2/(A_H - A_L)^2$, $\gamma = 1 - \alpha(2\theta - 1)$ *and*

$$g_1(\alpha, n) = 2\gamma^n + \frac{n(n+2)\gamma^n}{2 + n(n-1)\gamma^{n-1}}\left(4n + n(n-3)\gamma^{n-1} - 2(n+1)\ln\frac{1}{\gamma}\right)$$
$$g_2(\alpha, n) = 2\gamma^n + \frac{n\gamma^n}{2 + n(n-1)\gamma^{n-1}}\left(4n + n(n-3)\gamma^{n-1} - 2(n+1)\ln\frac{1}{\gamma}\right)$$

Then: (i) when $g_1(\widehat{\alpha}_n, n) > G_1(A_H, A_L, MC)$ *holds,* $\frac{d\overline{W}(\widehat{\alpha}_n,n)}{dn} < 0$*, that is, product market competition decreases total welfare; and*

(ii) when $g_2(\widehat{\alpha}_n, n) > G_1(A_H, A_L, MC)$ *holds,* $\frac{d\overline{CS}(\widehat{\alpha}_n,n)}{dn} < 0$*, that is, product market competition decreases consumer welfare.*

Lemma 3 characterizes when competition decreases real efficiency. First, note that the condition in Lemma 3 is non-empty. For example, this occurs when the price sensitivity b of demand is sufficiently high such that the probability of misallocation is large.[11]

[10] Using Equation (14), we can directly compute: $\frac{\partial \overline{W}(\widehat{\alpha}_n,n)}{\partial \widehat{\alpha}_n} = \frac{n^2(n+2)(2\theta-1)\widehat{\gamma}_n{}^{n-1}}{8b(n+1)^2}(A_H - A_L)^2 > 0$.

[11] Note that $\lim_{b\to\infty}\widehat{\alpha}_n = 0$. Then, we get the approximation $g_1(\widehat{\alpha}_n, n) = \frac{n^2(n+1)(n+2)}{n(n-1)+2} + 2 + O(n\widehat{\alpha}_n)$,

Second, Lemma 3 examines the role of market uncertainty $(A_H - A_L)$ and average profitability $(\bar{A} - MC)$ in shaping the efficiency effects of product market competition through feedback. Specifically, $G_1(A_H, A_L, MC)$ increases with average profitability and decreases with market uncertainty. Thus, when market uncertainty is high and average profitability low, the condition in Lemma 4.2(i) is more likely to hold, leading to a negative welfare effect from product market competition.

Third, the potential negative welfare effect depends on the probability of misallocation $(\widehat{\gamma}_n)^n$ through $g_1(\widehat{\alpha}_n, n)$. When the probability of misallocation is maximized ($\widehat{\gamma}_n = 1$), we estimate $g_1 = 2 + \frac{n^2(n+1)(n+2)}{2+n(n-1)}$. As $\widehat{\gamma}_n$ approaches zero, g_1 tends to zero. Thus, g_1 increases with the probability of misallocation or decreases with information production, although it is not strictly monotonic in either variable. This suggests that the negative welfare effect of competition ($g_1(\widehat{\alpha}_n, n) > G_1(A_H, A_L, MC)$) is more likely when the probability of misallocation is not too low, allowing feedback effects to generate enough gains in allocative efficiency when competition decreases. However, Section 4.2 points out that feedback effects may instead cause a loss in allocative efficiency. Such losses would reduce total welfare, consistent with the nonmonotonicity of $g_1(\widehat{\alpha}_n, n)$.

Since Lemma 3 involves the endogenous variable of information acquisition, we now provide a more direct result through constructive derivations.

Proposition 3 (Welfare-destructive Overcompetition)**.**

Consider a pair of positive integers (m, n) satisfying $\Phi(m) \geq 1$ and $n > N(m)$, where[12]

$$\Phi(m) = \left(1 + \frac{(A_H - A_L)^2(1 - (2 - 2\theta)^m)}{4(\overline{A} - MC)^2}\right) \times \frac{m(m+2)}{(m+1)^2}$$

$$N(m) = \frac{(m+1)^2}{(2-2\theta)\left(2 + (m-1)(2-2\theta)^{m-1}\right)} \geq m + 1$$

Then: $\overline{W}(\widehat{\alpha}_m, m) > \overline{W}(\widehat{\alpha}_n, n)$ holds for any $c \in [\bar{c}_n, \underline{c}_m)$ with $\bar{c}_n < \underline{c}_m$.

Denote $m_0 := \inf\{m \in \mathbb{N} : \Phi(m) \geq 1\} < \infty$. Proposition 3 shows that when the number of firms exceeds $N(m_0)$, the total welfare is strictly less than with m_0 firms.

Theorem 1 below directly follows from Proposition 3.

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where $O(\cdot)$ means "big O". Now suppose that $g(0, n) > G_1$, or equivalently, $\frac{(\bar{A}-MC)^2}{(A_H-A_L)^2} < \frac{n^2(n+1)(n+2)}{8(n(n-1)+2)}$. By continuity, for any $\widehat{\alpha}_n > 0$ sufficiently small, $g_1(\widehat{\alpha}_n, n) > G_1(A_H, A_L, MC)$ holds.

[12]Note that $\Phi(m) \geq 1$ is non-empty because $\lim_{m\to\infty} \Phi(m) = 1 + \frac{(A_H-A_L)^2}{4(\overline{A}-MC)^2} > 1$. Furthermore, since $\Phi(m)$ strictly increases in m, $\Phi(m_1) > \Phi(m_2)$ if $m_1 > m_2$.

Theorem 1. *Competition can reduce total welfare through informational feedback effects.*

Theorem 1 underscores the welfare-reducing effect of competition through information feedback. Specifically, when information production $\widehat{\alpha}$ is fixed, Equation (14) shows that increasing the number of firms always raises total welfare. Thus, Theorem 1 reveals that competition reduces welfare solely through the information production channel. Furthermore, for any positive integer m that satisfies $\Phi(m) \geq 1$, there exists a range of cost parameters c for which excessive competition lowers the total welfare when $n \geq N(m)$.

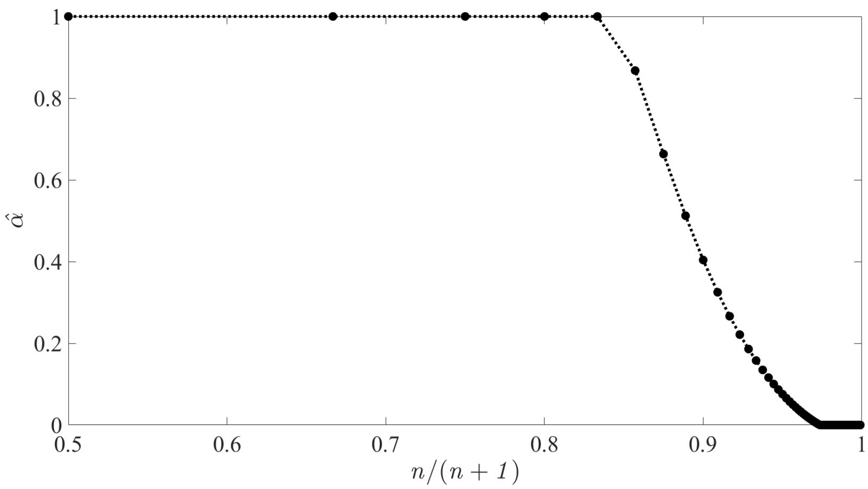

Figure 3: Product Competition and Information Production

Our main insight is illustrated in Figures 3 and 4.[13] First, Figure 3 shows how intensified competition affects information production incentives (Proposition 2). As competition increases ($n \uparrow$), information production transitions from full information ($\widehat{\alpha} = 1$), to partial information ($0 < \widehat{\alpha} < 1$), and ultimately to none ($\widehat{\alpha} = 0$). Second, Figure 4 illustrates the non-monotonic welfare effects of competition, with total welfare maximized at $n = 6$. Specifically: (i) for n small, the welfare increases as the market power declines; (ii) for n intermediate, the welfare decreases as the feedback effect dominates; and (iii) for n large, the welfare increases again as information production ceases, making the market power concentration channel dominant.

Interestingly, the interplay between Figure 3 and Figure 4 reveals two notable patterns that warrant closer examination. First, the decline in information production precedes the reduction in total welfare. Second, the observed non-monotonicity is primarily attributable to an interior solution in information production, rather than corner solutions. In addition,

[13]Baseline parameters are $\theta = 0.75$, $b = 1.5$, $A_H = 30$, $A_L = 10$, $c = 1.5$, and $MC = 3$, used throughout unless stated otherwise. See online Appendix B.3 for analogous results using US market data.

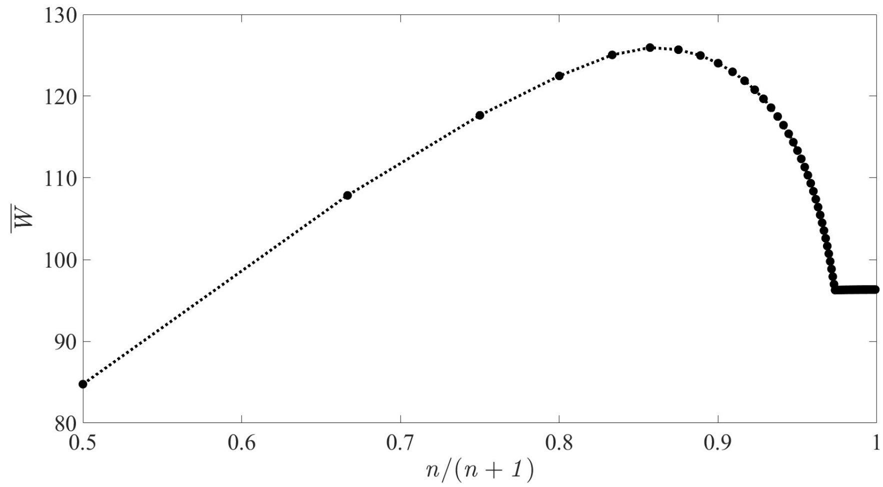

Figure 4: Product Competition and Total Welfare

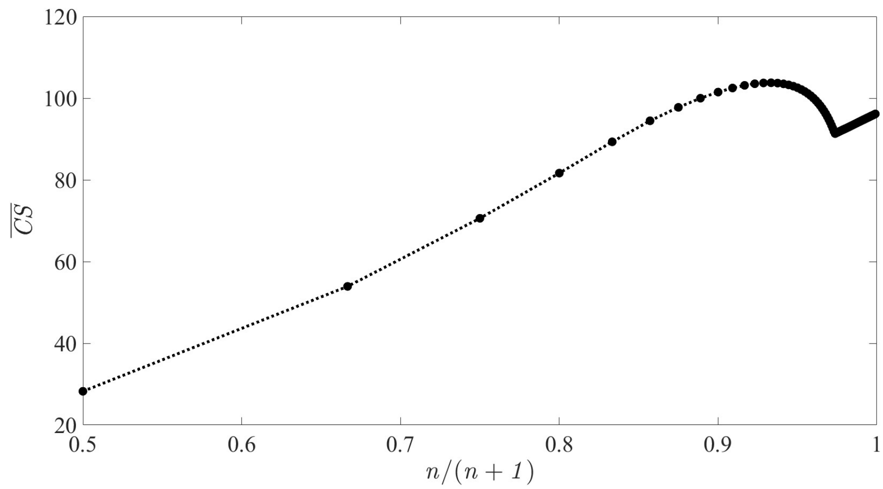

Figure 5: Product Competition and Consumer Surplus

Figure 5 illustrates a similar non-monotonic pattern in consumer surplus when we vary the number of firms n.[14]

Remark 1. *Under extreme parameter values, where low market uncertainty reduces the informational value of managerial learning, the stock market feedback effect may not overturn the positive link between competition and total welfare. Nonetheless, it can significantly shape the efficiency implications of firm competition, making it a crucial factor in regulating horizontal mergers. See online Appendix B.1 for a detailed discussion.*

[14] Specifically, in this numerical example, the consumer surplus increases first for $n \leq 14$, then decreases for $14 \leq n \leq 37$, and finally increases again for $n \geq 37$. Note that the consumer surplus is maximized at $n = 14$, rather than at $n = 6$.

4.4 Optimal Market Structure and Comparative Statics

This section examines the optimal market structure and performs comparative statics. Without feedback effects, the maximum total welfare is achieved as $n \to \infty$. However, with feedback effects, competition may reduce efficiency, and the maximum welfare may occur at a finite n^*, which we define as the optimal market structure.

Proposition 4 (Optimal Market Structure)**.** *The optimal market structure, n^*, can be non-monotonic in the information production cost c and the price sensitivity b.*

The non-monotonicity in Proposition 4 is driven by feedback effects and allocative efficiency gains. The negative relationship between competition and welfare results from the sensitivity - rather than the absolute level - of information production to changes in competition. When information costs are very high, no speculators acquire information, eliminating feedback effects. Conversely, when costs are very low, all speculators acquire information, making information production insensitive to competition. Thus, competition reduces welfare only for intermediate information costs where an interior equilibrium emerges.

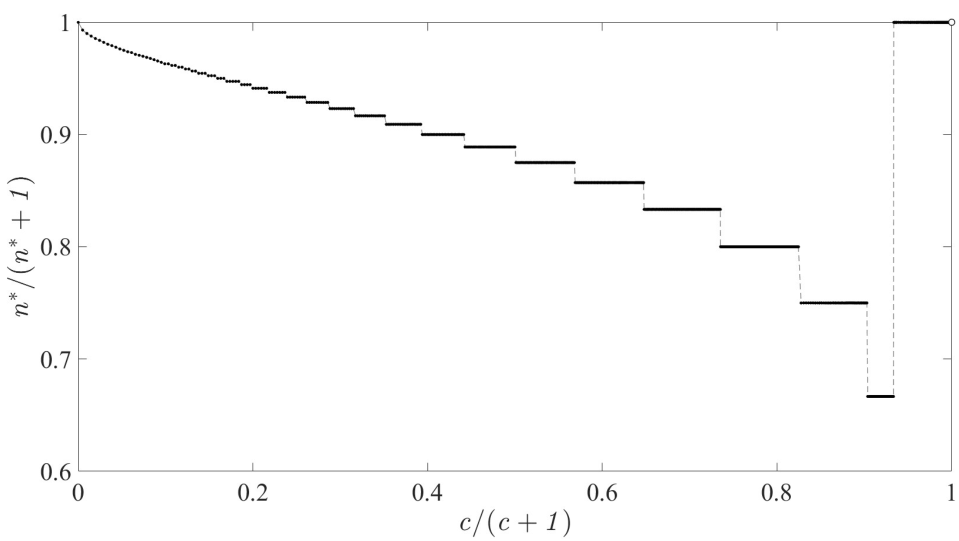

Figure 6: Optimal Market Structure n^*

Figure 6 illustrates the non-monotonic dependence of the optimal market structure n^* on information production cost c. As c decreases, n^* initially moves from perfect competition to a duopoly and then expands to three or more firms. In intermediate ranges of c, partial information production occurs, and fewer firms may dominate more firms in terms of welfare. For sufficiently low costs, most speculators become informed, making information production insensitive to changes in n and leading welfare to rise with increased competition. A similar pattern emerges for price sensitivity b (see online Appendix B.2).

Average profitability and market uncertainty. To better illustrate their economic intuition and implications, we discuss the role of average profitability and market uncertainty in shaping the link between competition and total welfare when $n^* < \infty$. Specifically, we use numerical methods to address the complexity of the auxiliary function $g_1(\alpha, n)$, complementing our earlier analytical results. Theoretical insights, including Lemma 3 and the following discussions in Section 4.3, provide guidance for the numerical analysis. We anticipate that a negative relationship between competition and total welfare is more likely to occur with high market uncertainty $(A_H - A_L)$ and low average profitability $\left(\bar{A} - MC\right)$. Meanwhile, by Equation (8) and Equation (11), these two factors also contribute to information production $\widehat{\alpha}$ in equilibrium. Define:

$$\Delta W_n := \overline{W}\left(\widehat{\alpha}_n, n\right) - \overline{W}\left(\widehat{\alpha}_{n-1}, n-1\right).$$

Obviously, a negative relationship between product market competition and total welfare ensues when $\Delta W_n < 0$ holds. We also focus on interior solutions of $\widehat{\alpha}_n$. Sensitivity analyses performed on a wide range of model parameter values have shown a similar pattern.

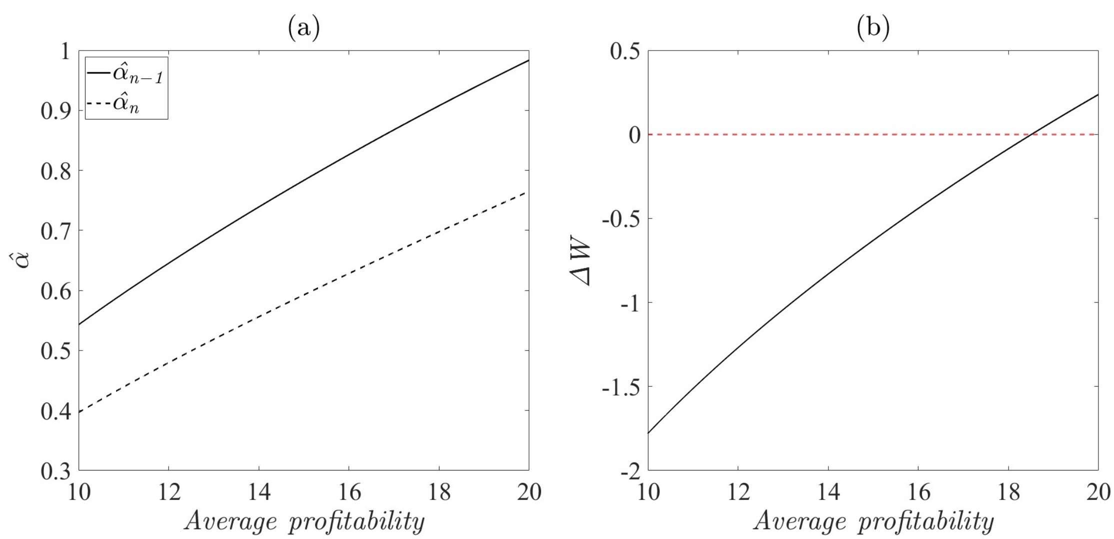

Figure 7: Average Profitability, Information Quality and Welfare.

Then we analyze the impact of average profitability $(\bar{A} - MC)$ on equilibrium information production $\widehat{\alpha}_n$ and total welfare ΔW_n. For this exercise, we fix the value of $(A_H - A_L)$ and other parameters. The results are plotted in Figure 7. We make three observations: First, Figure 7a shows that $\widehat{\alpha}_n$ is always lower than $\widehat{\alpha}_{n-1}$, which is consistent with the prediction of Proposition 2 that product market competition dampens the incentive for speculators to produce information. Second, both $\widehat{\alpha}_n$ and $\widehat{\alpha}_{n-1}$ increase strictly in average profitability, implying that higher profitability improves information acquisition. Third, Figure 7b shows that the welfare gain ΔW_n is smaller for a lower level of average profitability. In particular, when the average profitability is sufficiently low, ΔW_n can be negative, indicating that intensified competition decreases the total welfare. Note that this result coincides with our discussion following Lemma 3.

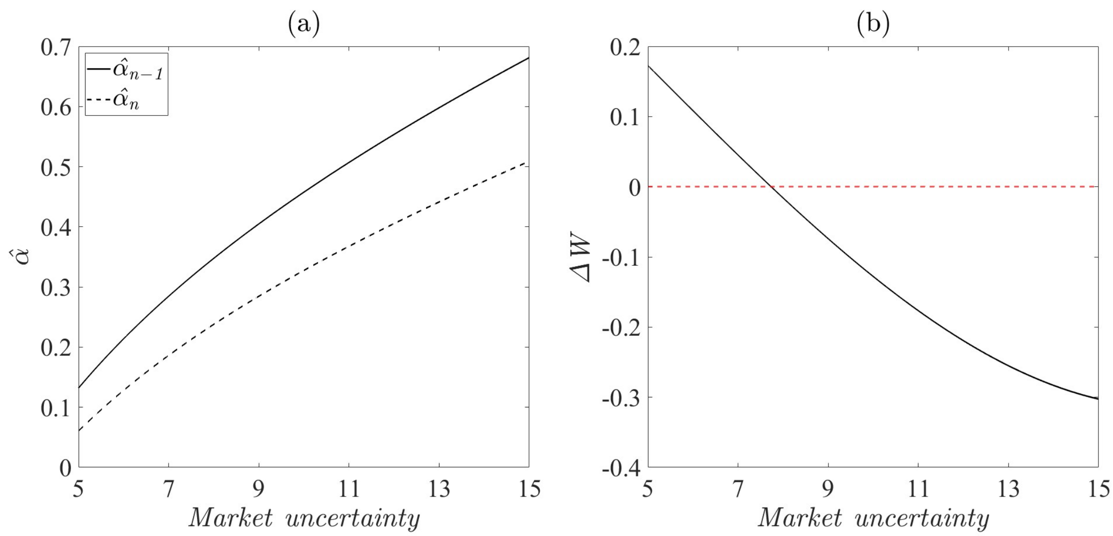

Figure 8: Market Uncertainty, Information Quality and Welfare.

Next, we investigate the effects of market uncertainty on $\widehat{\alpha}_n$ and ΔW_n by varying $(A_H - A_L)$ while keeping the average profitability $(\bar{A} - MC)$ and other parameters unchanged. These results are depicted in Figure 8. We make two observations: First, Figure 8a shows that both $\widehat{\alpha}_n$ and $\widehat{\alpha}_{n-1}$ increase as $(A_H - A_L)$ increases, which implies that increasing market uncertainty improves information production. Second, as shown in Figure 8b, competition can decrease total welfare when market uncertainty is high, despite the high incentive of information production (i.e., $\widehat{\alpha}$ is high).

This illustrates a sharp difference between average profitability and market uncertainty. Although both exhibit similar effects on information production, the welfare implications of competition diverge. Specifically, a negative relationship between competition and total welfare is more likely to occur when: (i) the average profitability is low; or (ii) the market uncertainty is high. To understand this divergence, we highlight two observations: First, an increase in average profitability directly increases total welfare, which reduces the relative impact of information production, while an increase in market uncertainty amplifies that of information production (see Equation (14)). Second, the negative link between competition and welfare depends on the relative gap, rather than the absolute intensity, in information production when the level of competition varies.

4.5 Implications for Horizontal Mergers

To better illustrate the empirical implications for horizontal mergers, we first compare a monopoly (i.e., $n = 1$) and a duopoly (i.e., $n = 2$) in perfectly symmetric Cournot competition.By Equation (14), the total welfare for a monopolist seller is given by:

$$\overline{W}\left(\widehat{\alpha}_1, 1\right) = \frac{3}{32b}\left(4\left(\bar{A} - MC\right)^2 + \left(1 - \widehat{\gamma}_1\right)\left(A_H - A_L\right)^2\right) \tag{17}$$

and that for two duopoly sellers are given by

$$\overline{W}\left(\widehat{\alpha}_2, 2\right) = \frac{1}{9b}\left(4\left(\bar{A} - MC\right)^2 + \left(1 - \left(\widehat{\gamma}_2\right)^2\right)\left(A_H - A_L\right)^2\right) \tag{18}$$

Obviously, if we fix the size of informed traders $\widehat{\alpha}_1 = \widehat{\alpha}_2$ (or equivalently $\widehat{\gamma}_1 = \widehat{\gamma}_2$) to shut down the information production channel, a duopoly market always outperforms a monopoly in total welfare. In other words, any regulatory action based on market concentration measures is well-founded. However, if we allow for endogenous information production, the above insight might not hold, as illustrated by Lemma 4 below.

Lemma 4 (Monopoly VS. Duopoly)**.**

Assume that $A_H > A_L = MC$*. Denote* $\kappa = (2\theta - 1)(A_H - A_L)^2/b$*.*

(i) When $\frac{\kappa}{12} \leq c < \frac{11}{108}\kappa$*, then* $\overline{W}\left(\widehat{\alpha}_1, 1\right) > \overline{W}\left(\widehat{\alpha}_2, 2\right)$*; and*

(ii) when $c \geq \frac{11}{108}\kappa$ *or* $c < \frac{(1-\theta)(2-\theta)\kappa}{9}$*, then* $\overline{W}\left(\widehat{\alpha}_1, 1\right) \leq \overline{W}\left(\widehat{\alpha}_2, 2\right)$*.*

We briefly comment on Lemma 4. First, a monopoly dominates a duopoly for an intermediate level of information production cost c. In Statement (i), a lower bound $c \geq \frac{\kappa}{12}$ is imposed to completely remove information production in a duopoly market (i.e., $\widehat{\alpha}_2 = 0$), while an upper bound $c < \frac{11\kappa}{108}$ ensues that the incentive to produce information is strong enough in a monopoly market (i.e., $\widehat{\alpha}_1 \uparrow$). Second, when information production is too cheap or too costly, the relative gap in information production is small, and thus a duopoly market is more efficient due to lowered market concentration.

Obviously, our theory differs sharply from the existing literature on merger analysis, which largely ignores the information efficiency of the stock market and often features a monotonic relationship between competition and total welfare in perfectly symmetric Cournot competition when all firms are equally efficient (see, e.g., Farrell and Shapiro, 1990). In contrast, even in the simplest case here, merging two competing and equally efficient firms into a monopolist can improve social welfare for an intermediate level of information production cost when market concentration significantly increases information production. This naturally arises when managerial learning from the stock market benefits production decisions in a feedback loop. Our theory highlights the importance of considering the interaction between the product market and the financial market in M&As regulations from an informational perspective.[15]

Remark 2 (Beyond Monopoly & Duopoly). *We can extend the analysis beyond two firms. Theorem 1 offers a framework for this analysis. Define* $m_0 := \inf\{m \in \mathbb{N} : \Phi(m) \geq 1\}$*. For* $n \geq N(m_0)$*, over-competition emerges in terms of total welfare within an intermediate range of information production costs, as it is strictly dominated by a market structure with* $n = m_0$*. Thus, reducing the number of firms to* $n < N(m_0)$ *can enhance total welfare, though the optimal number* n^* *requires numerical determination.*[16]

Furthermore, our treatment of M&As closely follow the spirit of Cournot competition in the long-run sense, differing from that of Nocke and Whinston (2022), where the post-merger HHI merely aggregates pre-merger market shares. Our analysis complements existing M&A frameworks by emphasizing the interplay between financial and product markets, alongside well-documented factors such as production efficiency asymmetries (Farrell and Shapiro, 1990), synergies (see, e.g., Maksimovic and Phillips, 2001), disclosure (Xiong and Yang, 2021), investment (Mermelstein et al., 2020; Motta and Tarantino, 2021), and innovation (Yi, 1999; Aghion et al., 2005; Segal and Whinston, 2007; Spulber, 2013).

A "calibrated" illustration. We present a numerical example to illustrate the welfare effects of a horizontal merger under the feedback effect. Although this is not intended as a formal calibration directly comparable to the US economy, it offers qualitative insights into

[15]While this non-monotonic relationship between competition and total welfare also appears in other studies on, the non-monotonicity there stems from some presumptions of anticompetitive effects such as cost synergies (see, e.g., Nocke and Whinston, 2022). We abstract away from those considerations to focus on the impact of informational feedback.

[16]The dominated structures $n \geq N(m)$ can also be chosen conditional on the information cost c.

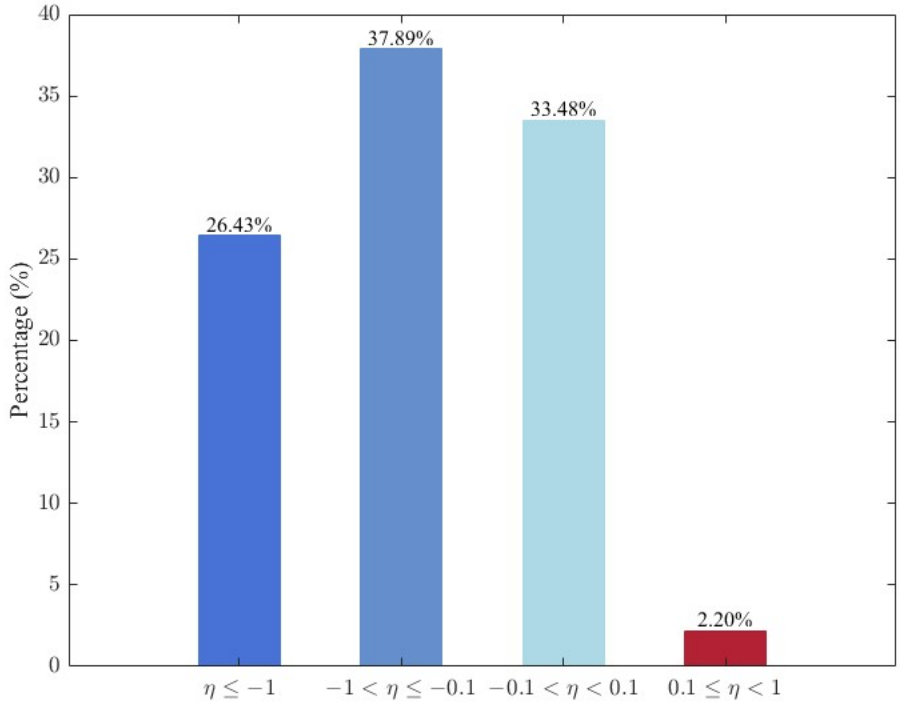

Figure 9: Estimation of η by industries

Notes: This histogram summarizes the estimation of η across industries, which are classified following Gu (2016) and Hou and Robinson (2006). The estimation is based on model parameters calibrated with US market data over 2000–2010. A negative value of η indicates that the welfare effect of a horizontal merger will be overestimated if the feedback effect is ignored. A positive value of η then suggests that the feedback effect augments the welfare effect of a horizontal merger.

the significance of feedback effects in assessing the economic implications of mergers.

Specifically, the welfare effect of a horizontal merger, both with and without feedback effects, can be expressed as $\overline{W}\left(\widehat{\alpha}_n, n\right) - \overline{W}\left(\widehat{\alpha}_{n-1}, n-1\right)$ and $\overline{W}(0, n) - \overline{W}(0, n-1)$. We then define the impact of informational feedback from the stock market on the welfare of horizontal mergers as:

$$\eta = \frac{\overline{W}\left(\widehat{\alpha}_n, n\right) - \overline{W}\left(\widehat{\alpha}_{n-1}, n-1\right)}{\overline{W}(0, n) - \overline{W}(0, n-1)} - 1. \tag{19}$$

Using US market data and the calibration method detailed in online Appendix B.3, we estimate model parameters and compute the corresponding values of η in all industries after excluding firms in the financial and utility industries, as well as industries with negative gross margins.

Figure 9 illustrates the industry-level distribution of η values. The key findings are as follows. On the one hand, in 64.32% of all industries, including the first two bars in Figure 9, the feedback effects of the stock market significantly weaken the welfare effect of horizontal mergers by more than 10%. Furthermore, in 26.43% of all industries, the impact of stock market feedback exceeds 100%, which implies that it completely reverses the welfare effects. On the other hand, in 2.20% of all industries, feedback effects amplify the welfare effect of mergers (referred to as the augmentation effect).

Overall, these results highlight that feedback effects from the stock market constitute a critical factor in analyzing the welfare impact of horizontal mergers and the efficiency of market competition. Ignoring these effects can lead to misinterpretations of merger outcomes.

5 Further Discussions

5.1 Cross-Asset Trading

Although standard in the literature (see, e.g., Foucault and Frésard, 2014, 2019), bounded asset positions ($x_k^i \in [-1,1]$) in our baseline model may not be as harmless as in other settings: If the total product market size is stable, with an increase in the number of firms, the size and, consequently, the equity value of each firm decrease. Therefore, the dollar value of the maximum trade size could decrease in n, and thus the incentive to acquire information might mechanically decrease. To address this concern and show robustness, we now allow cross-asset trading, in which a fraction of speculators can trade all stocks. All baseline findings continue to hold.

Specifically, we consider an economy with $n \geq 2$ identical firms competing in quantities and a stock exchange, which is populated with four types of investors, including: (i) a mass $\lambda \in [0,1]$ of risk-neutral L-traders $k \in [0,\lambda]$, who choose whether to acquire a costly signal m_k at a cost $c_L > 0$, and trade all stock shares $y_k^i \in [-1,1]$ for all i; (ii) a mass $1-\lambda$ of risk-netural S-traders $k \in [0, 1-\lambda]$ for each stock i, who choose whether to acquire a costly signal m_k^i at a cost $c_S > 0$ and only trade shares $x_k^i \in [-1,1]$ for the ith stock. (iii) liquidity traders with aggregate demand z_i, uniformly distributed over $[-1,1]$, for each firm i, and (iv) risk-neutral market makers who set prices to clear each stock.

Let $y_i = \int_0^{\alpha_L} y_k^i dk$ and $x_i = \int_0^{\alpha_{i,S}} x_k^i dk$ denote the aggregate demand for stock i by L- and S-traders. Recall that the aggregate order submitted by liquidity traders is z_i. Thus, the total order flow f_i for the ith stock is then given by: $f_i = x_i + y_i + z_i$. As in Goldstein et al. (2014), we assume that $c_L \leq c_S$, i.e., an L-trader has a relatively lower cost of information production.[17] For ease of reference, let α_L and $\alpha_{i,S}$ denote the measure of informed L-traders and that of informed S-traders for the ith firm. Define $\boldsymbol{\alpha} := (\alpha_L, \alpha_{1,S}, \cdots, \alpha_{n,S})$. All other features of the model are the same. Note that when $\lambda = 0$, it reduces to the baseline setup.

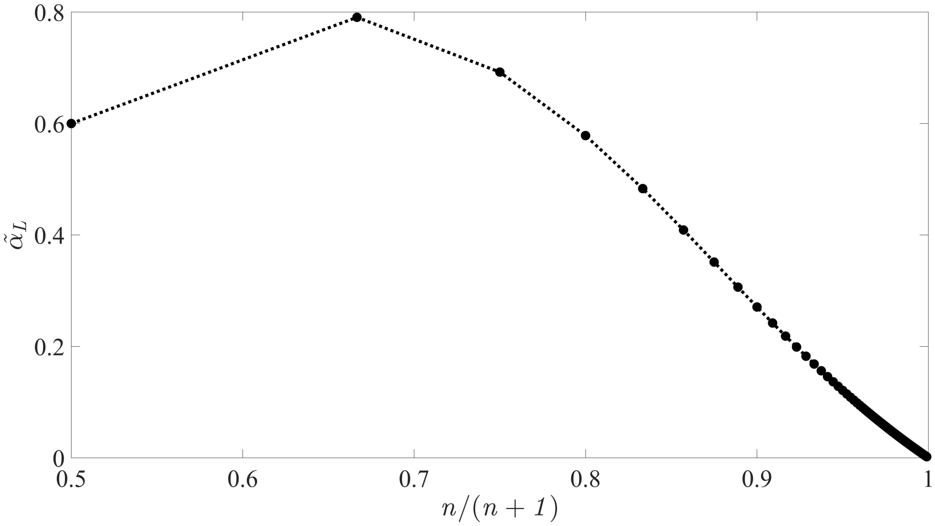

Figure 10: Trading Opportunities & (Non-monotonic) Information Production

We briefly summarize the key insights, while the equilibrium analysis can be found in online Appendix B.4. First, L-traders have a stronger incentive to acquire information than S-traders, given that $c_L \leq c_S$. Actually, the incentive for L-traders to acquire information can even increase in the number of firms n, which differs sharply from S-traders for whom the incentive for information acquisition is always maximized in a monopoly. This complexity is illustrated in Figure 10.[18] In particular, when we move from a monopoly ($n = 1$) to a duopoly ($n = 2$), the size of the informed L-traders $\widetilde{\alpha}_L$ first increases and then decreases.[19]

Second, our baseline result remains valid in the presence of L-traders, because the incentive for information production for L-traders will drop quickly after achieving its maximum level, and thus a negative relationship between competition and total welfare ensues.

5.2 Cross-Asset Learning

In the baseline model, we assume that the market maker of the ith firm does not observe the order flow of the other firms. Therefore, there may be arbitrage opportunities between

[17] To be precise, Goldstein et al. (2014) sets $c_S > c_L = 0$, i.e., an L-trader costlessly observes a signal.

[18] Parameters used for the extended model with cross-asset trading are: $\lambda = 0.8$, $\theta = 0.75$, $b = 3.5$, $A_H = 20$, $A_L = 10$, $MC = 9$, and $c_L = c_S = 1.5$.

[19] Vives (1985) shows that the profit of competing firms vanishes at a speed order of $1/n$. When multiplied by the number of firms n, the trading profits for L-traders can be non-monotonicity in n. We term this the "trading opportunity effect" in cross-asset trading.

competing firms. This section removes this restriction and considers cross-asset learning, which refers to the possibility that market makers observe the order flow in all stocks before setting the price (see, e.g., Pasquariello and Vega, 2015; Foucault and Frésard, 2019). Specifically, we modify the more general setup in Section 5.1 by allowing for cross-asset learning, i.e., the information set for market makers is $\Omega = \{f_1, \cdots, f_n\}$. Again, as in Kyle (1985), risk-neutral market makers absorb excess order flow and break even only in expectation. Thus, the stock price of the ith firm is given by $s_i(\Omega) = \mathbb{E}[V_i|\Omega]$.

Here, we briefly discuss the main results with cross-asset learning, and delegate the formal analysis to online Appendix B.5. First, the baseline result holds in the presence of cross-asset learning when there are only S-traders. Intuitively, cross-asset learning empowers market makers, reducing trading profits for speculators, except for the special case with a monopoly. This in turn makes the trading profits more sensitive to the change in the number of competing firms when it is small. Thus, the information feedback channel is strengthened.

Second, the non-monotonicity can also appear when the cost of information production is small such that all L-traders choose to acquire information. Note that L-traders have a stronger incentive to acquire information compared to S-traders. Cross-asset trading makes S-traders more prone to competition compared to L-traders, and thus L-traders may crowd out S-traders due to their trading opportunities.

Third, total welfare can strictly increase with the number of firms n in the presence of cross-asset learning when S-traders are totally absent. In practice, however, markets are unlikely to consist solely of L-traders, as segmentation due to various frictions is common; see, e.g., Goldstein et al. (2014) for real-world examples of market segmentation. Moreover, even in markets with only L-traders, the efficiency implications of firm competition can still be significantly influenced by informational feedback from the stock market (though not to the extent of creating a non-monotonic relationship between competition and welfare). This feedback effect often exacerbates allocative efficiency losses as product market competition weakens, amplifying the welfare losses associated with market power concentration. Thus, the feedback effect remains a critical factor to consider in regulating horizontal mergers, even in the absence of S-traders. For a detailed discussion on the divergent impacts of cross-asset learning on L-traders and S-traders, see online Appendix B.5.

5.3 Investor Welfare

Investor welfare, especially that of liquidity traders, is largely missing from the total welfare defined in Equation (14), which essentially captures the welfare of the product market, including both the consumer surplus and the producers' surplus. We now show that our theoretical insights still hold when we include investor welfare in the calculation of total welfare. Recall that: (1) market makers always break even in expectation; (2) informed speculators incur acquisition costs but earn positive trading profits; (3) liquidity traders incur trading losses but enjoy liquidity benefits; and (4) informed speculators' trading profits equal liquidity traders' trading losses. Although liquidity benefits are conceptually endogenous, most papers treat them and liquidity trading as completely exogenous. The total cost of information acquisition varies with the size of informed speculators α, and given that we focus on the benefits of information, the cost of information acquisition should not be overlooked.

Specifically, let $B(n)$ denote the aggregate benefit of liquidity trading. Thus, total welfare $\overline{W}_{PF}$, including both product market welfare and investor welfare, can be measured as:

$$\overline{W}_{PF} = \overline{W} - n * \widehat{\alpha}_n * c + B(n) \tag{20}$$

where $\overline{W}(\widehat{\alpha}_n, n)$ is given by Equation (14).

When the aggregate benefits of liquidity trading are exogenously fixed (i.e., $B(n) = B_0$ for some non-negative constant B_0), a non-monotonic relationship between product competition and total welfare can arise, and the optimal market structure features a finite number of firms. Such non-monotonicities may manifest under other specifications if the aggregate benefits of liquidity trading are proportional to the number of stocks, although the optimal market structure might approach perfect competition when the benefits of liquidity trading become dominant. Online Appendix B.6 contains a formal analysis.

5.4 Discount Rates

In our primary analysis, we have not accounted for the effects of discounting. However, as Cochrane (2011) highlights, discount rates, rather than cash flows, may drive movements in stock prices, at least at the aggregate level. Given that variations in industrial competition can influence discount rates (Dou et al., 2021), incorporating discounting into the evaluation of firm value and stock prices could potentially alter our findings. To address this, we extend our baseline model to explore the implications of discounting.

Let $r_n \geq 0$ denote the discount rate when n symmetric firms compete in the industry. Then, the expected firm value given in Equation (2) can be rewritten as:

$$V_i(q_i) = \frac{1}{1+r_n}\mathbb{E}\left[TP_i(q_i) \mid \mathrm{F}_m\right]$$

Note that the profit function $TP_i(q_i)$ is linear in the parameters A, b and MC, as shown in Equation (1). Thus, introducing discounting into the model is equivalent to replacing the original parameters (A, b, MC) with a set of new parameters (A', b', MC'), where

$$A'_\omega = \frac{A_\omega}{1+r_n}, \quad b' = \frac{b}{1+r_n}, \quad \text{and} \quad MC' = \frac{MC}{1+r_n}$$

Furthermore, the linearity implies that the baseline results in Section 3 can be obtained using (A', b', MC'). We now discuss the relationship between competition and discount rates and how it affects our results in Section 4. First, we assume that the discount rate r_n strictly increases in n (that is, $\frac{\partial r_n}{\partial n} > 0$) because increased competition can erode profitability and increase risk. This assumption is consistent with the existing literature that documents a positive correlation between competition and discount rates (Dou et al., 2021). We can use the chain rule of differentiation to get: $\frac{\partial \Pi'(n,\alpha)}{\partial n} = \frac{1}{1+r_n}\frac{\partial \Pi(n,\alpha)}{\partial n} - \frac{1}{(1+r_n)^2}\frac{\partial r_n}{\partial n} < \frac{1}{1+r_n}\frac{\partial \Pi(n,\alpha)}{\partial n}$ and $\frac{\partial \Pi'(n,\alpha)}{\partial \alpha} = \frac{1}{1+r_n}\frac{\partial \Pi(n,\alpha)}{\partial \alpha}$, which further implies:

$$\frac{\partial \widehat{\alpha}'_n}{\partial n} < \frac{\partial \widehat{\alpha}_n}{\partial n} < 0$$

Thus, when discounting is considered, increased competition discourages speculators from acquiring information. More importantly, discounting can exacerbate this negative impact of competition on information production. Consequently, we can reasonably anticipate that our main result will not only remain valid but may also be strengthened by the compounding effects of discounting. Specifically, reduced information production in the stock market, driven by intensified competition, could significantly decrease the allocative efficiency of the real economy, potentially leading to a negative relationship between competition and real efficiency due to feedback effects.

5.5 Dynamic Trading

Most existing studies focus on a static framework when modeling Cournot competition and feedback effects, as incorporating dynamic trading and competition can rapidly render the model intractable (Edmans et al., 2015; Goldstein and Yang, 2019; Lin et al., 2019). Consequently, we only provide an informal exploration of how our main results might be affected in a dynamic setting.

In general, introducing multiple rounds of trading creates opportunities for market manipulation, as the feedback effect from the stock market incentivizes speculators to influence stock prices (Edmans et al., 2015; Goldstein and Guembel, 2008). Specifically, uninformed traders may profit from selling the stock when feedback effects are present, partly because their trading distorts the information content of stock prices and misleads the firm's investment decisions. Consequently, we may expect that market manipulation, stemming from dynamic trading opportunities, could influence our main results by altering the informativeness of stock prices.

However, we argue that manipulation is more likely to occur in the stock trading of small firms rather than large firms. For instance, stocks characterized by high illiquidity and significant information asymmetry are more susceptible to manipulation (Comerton-Forde and Putniņš, 2014), and small-cap stocks typically exhibit low liquidity and limited transparency (Banz, 1981; Acharya and Pedersen, 2005). The reasoning is as follows. First, intensified competition reduces the size of firms, which in turn increases the potential for market manipulation. Second, information distortion caused by market manipulation can lead to a loss in real efficiency through feedback effects. As a result, our main findings should remain valid, and dynamic trading opportunities can further amplify the negative impact of competition on welfare by further suppressing the informativeness of stock prices when competition intensifies.

5.6 Additional Robustness Analysis

We discuss the robustness of our main results in three additional extensions, including the cost of information acquisition, the risk attitude of speculators, and firm heterogeneity.

First, our results depend on how product market competition affects speculators' costs of information acquisition. If increased competition raises these costs, reducing competition (e.g., via horizontal mergers) would encourage greater information production in the stock market and generate gains in allocative efficiency through the feedback mechanism. In contrast, if intensified competition reduces these information costs, horizontal mergers would suppress information production and harm allocative efficiency. However, empirical findings by Farboodi et al. (2022) indicate that information production is more active among larger firms. Since intensified competition shrinks the size of firms, information acquisition costs likely increase with competition. Hence, horizontal mergers—by decreasing competition and increasing firm size—should lower these costs, increase information production, and strengthen the feedback effect. Consequently, horizontal mergers are more likely to produce positive welfare effects when this feedback mechanism is present.

Second, traders and speculators are often risk-averse in practice, but our main findings remain valid. Increased competition raises firms' risks, discouraging risk-averse speculators from entering the market. This further reduces information production compared to the risk-neutral scenario, significantly harming real efficiency.

Third, we focus on symmetric Cournot competition and abstract from firm heterogeneity and synergies typically emphasized in merger analyses, where welfare effects depend on balancing market concentration (higher prices due to reduced competition) against operating efficiencies (cost reductions from synergies). For example, merging firms with complementary strengths, such as lower production costs and superior distribution, can create synergies that improve efficiency. Similarly, synergies can also be achieved through shared technologies or improved management practices. However, the main insights should extend to scenarios with firm heterogeneity and synergies. After a horizontal merger, reduced competition improves information production in the stock market. Since managers often misestimate market conditions, more informative stock prices help them correct biases and improve decisions. Therefore, stock market feedback provides an additional important channel that affects the welfare implications of horizontal mergers.

6 Conclusion

By incorporating information production and learning into a standard Cournot game, we analyze the interaction between product market competition and informational feedback in financial markets. Although intensified competition can reduce the concentration of market power and enhance the economic efficiency in production, it also reduces the incentives for speculators to acquire proprietary information on firms' market prospects. Consequently, a novel trade-off between economic efficiency and informational efficiency emerges endogenously when production decisions depend on the information conveyed in stock prices. Intensified product market competition can discourage information production in the stock market and generate losses in allocative efficiency through feedback effects, thus impacting the positive welfare effects of competition on real efficiency. When the feedback effect of stock prices is sufficiently strong, a negative relationship between product market competition and total welfare can even arise. Our model provides new insights for antitrust regulations in horizontal mergers, and a guidance for future studies exploring the intersection of financial market efficiency and product market competition.

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Appendix

A Proofs of Lemmas and Propositions

A.1 Proof of Lemma 1

Proof. We first compute the beliefs of the market makers. Recall that the total order flow for the ith stock is $f_i = \alpha_i(2\theta - 1) * (\mathbb{1}(\{\omega = H\}) - \mathbb{1}(\{\omega = L\})) + z_i$.[20] Denote $\gamma_i = 1 - \alpha_i(2\theta - 1)$. Note that condition $f_i > \gamma_i$ contradicts the event that $\omega = L$ because: (1) $f_i = z_i + x_i$ by definition; (2) $x_i = -\alpha_i(2\theta - 1)$ if $\omega = L$ by the law of large numbers; and (3) $z_i \leq 1$. Conversely, when $z_i > \gamma_i - \alpha_i(2\theta - 1)$ and $\omega = H$, then $f_i > \gamma_i$. Therefore, the aggregate order flow f_i is a sufficient statistic to update the beliefs of the market makers. In summary, if the aggregate order flow satisfies $f_i > \gamma_i$, it can be inferred that $\omega = H$. Similarly, if the aggregate order flow of stock i is $f_i < -\gamma_i$, the market makers will infer that $\omega = L$. Furthermore, when the aggregate order flow satisfies $f_i \in (-\gamma_i, \gamma_i)$, an application of the Bayes rule implies that

$$\Pr\left(\omega = H \mid f_i \in (-\gamma_i, \gamma_i)\right) = \frac{\Pr(\omega = H)\Pr\left(f_i \in (-\gamma_i, \gamma_i) \mid \omega = H\right)}{\Pr\left(f_i \in (-\gamma_i, \gamma_i)\right)} = \frac{1}{2}$$

because $\Pr\left(f_i \in (-\gamma_i, \gamma_i) \mid \omega = H\right) = \Pr\left(-\gamma_i - \alpha_i(2\theta - 1) \leq z_i \leq \gamma_i - \alpha_i(2\theta - 1)\right) = \gamma_i$ and $\Pr\left(f_i \in (-\gamma_i, \gamma_i)\right) = \Pr\left(f_i \in (-\gamma_i, \gamma_i), \omega = H\right) + \Pr\left(f_i \in (-\gamma_i, \gamma_i), \omega = L\right) = \gamma_i$. This also means that an order flow such that $f_i \in [-\gamma_i, \gamma_i]$ is uninformative.

Second, we analyze the belief updating rule for the ith manager, given the equilibrium prices $\{s_i(f_i)\}_{1 \leq i \leq n}$. Specifically, when $s_i(f_i) = s_H$ is observed, the manager i infers that $f_i > \gamma_i$ and thus $\omega = H$, which is exactly the reason for the market makers. Similarly, when $s_i(f_i) = s_L$ is observed, it can be inferred that $f_i < -\gamma_i$ and thus $\omega = L$. Finally, when $s_i(f_i) = s_M^i$, it must be the case that $f_i \in (-\gamma_i, \gamma_i)$, implying that the ith firm stock price is not informative about the market prospects. The ith manager depends on all other firms' stock prices to infer about the state, and there are three cases, including: (i) there exists some $j \neq i$ such that $s_j = s_H$, then again $f_j > \gamma_j$ and thus $\omega = H$; (ii) if there exists some $j \neq i$ such that $s_j = s_L$, then $f_j < -\gamma_j$ and thus $\omega = L$; (iii) if for all $j \neq i$ such that $s_j = s_M^j$, then it can be inferred that all stock prices are uninformative.

Next, we analyze the ith firm's production strategy, given the manager's posterior belief on the state ω after observing stock prices. Let θ_m be the posterior probability of $\omega = H$. Then, the ith manager's problem is to choose the quantity q_i to maximize:

$$V_i(q_i) = \mathbb{E}\left[TP_i(q_i) \mid \theta_m\right] = q_i\left(A_m - b(q_i + q_{-i}) - MC\right) \tag{A.1}$$

where $A_m = \mathbb{E}[\tilde{A}|\mathcal{F}_m] = \theta_m A_H + (1 - \theta_m) A_L$ is the expected value of the market prospect A conditional on posterior belief. From Equation (A.1), we know that $V_i(q_i)$ is concave in q_i, and thus $q_i^*(q_{-i}) = \frac{1}{2b}(A_m - bq_{-i} - MC)$. Given a common posterior belief updating rule, we can invoke $q_i = q_j$ for any $i \neq j$. Therefore, $q_i^* = \frac{A_m - MC}{(n+1)b}$.

[20] $\mathbb{1}(\{x \in A\})$ is an indicator function that equals one only when $x \in A$ holds, and equals zero otherwise.

Denote $q_H = \frac{A_H - MC}{(n+1)b}, q_L = \frac{A_L - MC}{(n+1)b}$, and $\beta_i = \prod_{j \neq i} \gamma_j$. Then, combining the belief updating rule of the common posterior, we conclude: (1) if $s_j = s_H$ for some j, then $\theta_m = 1$, $A_m = A_H$ and $q_i^* = q_H$; (2) if $s_j = s_L$ for some j, then $\theta_m = 0$, $A_m = A_L$ and $q_i^* = q_L$; and (3) if $s_j = s_M^j$ for all $1 \leq j \leq n$, then $\theta_m = \frac{1}{2}$, $A_m = \bar{A}$ and $q_i^* = q_M$.

We now check that the stock price rule $s_i(f_i)$ in Equation (6) satisfies condition (4). First, when the total order flow of the ith stock satisfies $f_i > \gamma_i$, then $\omega = H$, and thus $q_i^* = q_H$. By Equations (1) and (2), $\mathbb{E}\left[V_i\left(q_i^*\right) \mid f_i\right] = \frac{(A_H - MC)^2}{(n+1)^2 b}$, which is equal to s_H. Thus, condition (4) is satisfied when $f_i > \gamma_i$. Second, when the total order flow satisfies $f_i < -\gamma_i$, the net demand for the ith stock reveals that $\omega = L$, and thus $q_i^* = q_L$. Hence, $\mathbb{E}\left[V_i\left(q_i^*\right) \mid f_i\right] = \frac{(A_L - MC)^2}{(n+1)^2 b}$ for $f_i < -\gamma_i$, which is equal to s_L. Thus, for $f_i < -\gamma_i$, condition (4) is satisfied.

Third, when $f_i \in (-\gamma_i, \gamma_i)$, the investor demand for the ith stock is not informative about the state, i.e., $\Pr\left(\omega = H \mid f_i \in (-\gamma_i, \gamma_i)\right) = \frac{1}{2}$. Furthermore, by the argument of common posterior belief above, the manager i will produce q_H if $s_j = s_H$ for some $j \neq i$, produce q_L if $s_j = s_L$ for some $j \neq i$, and produce q_M if $s_j = s_M^j$ for all $j \neq i$. Thus, given that $f_i \in (-\gamma_i, \gamma_i)$ and $\exists j \neq i : s_j = s_H$, the ith firm's total profit at time $t = 1$ from producing q_H is

$$TP_H = \frac{(A_H - MC)^2}{(n+1)^2 b}$$

When $f_i \in (-\gamma_i, \gamma_i)$ and $\exists j \neq i : s_j = s_L$, firm i 's total profit from producing q_L is

$$TP_L = \frac{(A_L - MC)^2}{(n+1)^2 b}.$$

When $f_i \in (-\gamma_i, \gamma_i)$ and $s_j = s_M^j$ for $\forall j \neq i$, we deduce that: (1) if $\omega = H$, firm i 's total profit in $t = 1$ from producing q_M is

$$TP_{MH} = \frac{(n+1)\left(\bar{A} - MC\right)\left(A_H - MC\right) - n\left(\bar{A} - MC\right)^2}{(n+1)^2 b};$$

and (2) if $\omega = L$, firm i 's total profit in $t = 1$ from producing q_M is

$$TP_{ML} = \frac{(n+1)\left(\bar{A} - MC\right)\left(A_L - MC\right) - n\left(\bar{A} - MC\right)^2}{(n+1)^2 b}.$$

Furthermore, by Equation (2), we obtain the following.

$$\begin{aligned}
\mathbb{E}\left[V_i\left(q_i^*\right) \mid f_i \in (-\gamma_i, \gamma_i)\right] \quad = \quad & \Pr\left(\exists j \neq i : s_j = s_H \mid f_i \in (-\gamma_i, \gamma_i)\right) \times TP_H \\
+ \quad & \Pr\left(\exists j \neq i : s_j = s_L \mid f_i \in (-\gamma_i, \gamma_i)\right) \times TP_L \\
+ \quad & \Pr\left(\forall j \neq i : s_j = s_M^j, \omega = H \mid f_i \in (-\gamma_i, \gamma_i)\right) \times TP_{MH} \\
+ \quad & \Pr\left(\forall j \neq i : s_j = s_M^j, \omega = L \mid f_i \in (-\gamma_i, \gamma_i)\right) \times TP_{ML}.
\end{aligned}$$

To compute $\mathbb{E}\left[V_i\left(q_i^*\right) | f_i \in (-\gamma_i, \gamma_i)\right]$, we first calculate the conditional probabilities. Applying the Bayes rule, we get:

$$\Pr\left(\exists j \neq i : s_j = s_H \mid f_i \in (-\gamma_i, \gamma_i)\right) = \frac{\Pr\left(\exists j \neq i : s_j = s_H, f_i \in (-\gamma_i, \gamma_i)\right)}{\Pr\left(f_i \in (-\gamma_i, \gamma_i)\right)}. \tag{A.2}$$

Using the law of total probability, we have

$$\begin{aligned}\Pr\left(\exists j \neq i : s_j = s_H, f_i \in (-\gamma_i, \gamma_i)\right) &= \Pr\left(\exists j \neq i : s_j = s_H, f_i \in (-\gamma_i, \gamma_i), \omega = H\right) \\ &+ \Pr\left(\exists j \neq i : s_j = s_H, f_i \in (-\gamma_i, \gamma_i), \omega = L\right)\end{aligned}$$

Note that $\Pr(\exists j \neq i : s_j = s_H, f_i \in (-\gamma_i, \gamma_i), \omega = L) = 0$ and that

$$\begin{aligned}\Pr\left(\exists j \neq i : s_j = s_H, f_i \in (-\gamma_i, \gamma_i), \omega = H\right) &= \Pr(\omega = H) \times \Pr\left(f_i \in (-\gamma_i, \gamma_i) \mid \omega = H\right) \\ &\times \Pr\left(\exists j \neq i : s_j = s_H \mid f_i \in (-\gamma_i, \gamma_i), \omega = H\right) = \frac{1}{2}(1-\beta_i)\gamma_i\end{aligned}$$

Thus, $\Pr\left(\exists j \neq i : s_j = s_H, f_i \in (-\gamma_i, \gamma_i)\right) = \frac{1}{2}(1-\beta_i)\gamma_i$.

Plugging this into Equation (A.2), we obtain: $\Pr\left(\exists j \neq i : s_j = s_H \mid f_i \in (-\gamma_i, \gamma_i)\right) = \frac{1}{2}(1-\beta_i)$.

Analogously, we can show: $\Pr\left(\exists j \neq i : s_j = s^L \mid f_i \in (-\gamma_i, \gamma_i)\right) = \frac{1}{2}(1-\beta_i)$ and

$$\begin{aligned}&\Pr(\forall j \neq i : s_j = s_M^j, \omega = H | f_i \in (-\gamma_i, \gamma_i)) \\ &\qquad = \Pr\left(\forall j \neq i : s_j = s_M^j, \omega = L \mid f_i \in (-\gamma_i, \gamma_i)\right) = \frac{1}{2}\beta_i\end{aligned}$$

Finally, plugging in these conditional probabilities, we have:

$$\mathbb{E}\left[V_i\left(q_i^*\right) \mid f_i \in (-\gamma_i, \gamma_i)\right] = \frac{2\left((A_H - MC)^2 + (A_L - MC)^2\right) - \beta_i\left(A_H - A_L\right)^2}{4(n+1)^2 b}$$

which is equal to s_M^i. Therefore, condition (4) is satisfied for $f_i \in [-\gamma_i, \gamma_i]$. The proof concludes. □

A.2 Proof of Lemma 2

Proof. Let $\Pi_i\left(x_k^i, m_k^i\right)$ be the expected profit of the speculator k who trades $x_k^i \in [-1, 1]$ shares of the ith firm when his signal is m_k^i, and let V_2^i be the market value of the ith firm at $t = 1$. Since each speculator is risk neutral and a price taker in the stock market, speculators will trade the maximum size possible if they acquire information, i.e., $x_k^i = \pm 1$.

First, consider an informed speculator who observes $m_k^i = H$. If he buys the asset, his expected profit is $\Pi_k^i(+1, H) = \mathbb{E}\left[V_2^i - s_i\left(f_i\right) \mid m_k^i = H, x_k^i = 1\right]$.

From the proof of Lemma 1, firm i 's value at $t = 1$ is

$$V_2^i = \begin{cases} TP_H & \text{if } \exists j \in \{1, \ldots, n\} \text{ such that } s_j = s_H; \\ TP_{MH} & \text{if } \omega = H \ \& \ s_j = s_M^j, \forall j \in \{1, \ldots, n\}; \\ TP_L & \text{if } \exists j \in \{1, \ldots, n\} \text{ such that } s_j = s_L; \\ TP_{ML} & \text{if } \omega = L \ \& \ s_j = s_M^j, \forall j \in \{1, \ldots, n\}. \end{cases} \tag{A.3}$$

Thus, using Equation (A.3), we can calculate $\Pi_i(+1,H)$ as follows:

$$\begin{aligned}
\Pi_i(+1,H) = &\Pr\left(\omega = H, f_i > \gamma_i \mid m_k^i = H\right) \times (TP_H - s_H) \\
&+ \Pr\left(\omega = L, f_i < -\gamma_i \mid m_k^i = H\right) \times (TP_L - s_L) \\
&+ \Pr\left(\omega = H, f_i \in (-\gamma_i, \gamma_i), \exists j \neq i : s_j = s_H \mid m_k^i = H\right) \times \left(TP_H - s_M^i\right) \\
&+ \Pr\left(\omega = H, f_i \in (-\gamma_i, \gamma_i), \forall j \neq i : s_j = s_M^j \mid m_k^i = H\right) \times \left(TP_{MH} - s_M^i\right) \\
&+ \Pr\left(\omega = L, f_i \in (-\gamma_i, \gamma_i), \exists j \neq i : s_j = s_L \mid m_k^i = H\right) \times \left(TP_L - s_M^i\right) \\
&+ \Pr(\omega = L, f_i \in (-\gamma_i, \gamma_i), \forall j \neq i : s_j = s_M^j \mid m_k^i = H\Big) \times \left(TP_{ML} - s_M^i\right).
\end{aligned}$$

Since $s^H = TP_H$ and $s^L = TP_L$, we can rewrite the expression of $\Pi_i(+1,H)$ as:

$$\begin{aligned}
\Pi_i(+1,H) &= \Pr\left(\omega = H, f_i \in (-\gamma_i, \gamma_i), \exists j \neq i : s_j = s_H \mid m_k^i = H\right) \times \left(TP_H - s_M^i\right) \\
&+ \Pr\left(\omega = H, f_i \in (-\gamma_i, \gamma_i), \forall j \neq i : s_j = s_M^j \mid m_k^i = H\right) \times \left(TP_{MH} - s_M^i\right) \\
&+ \Pr\left(\omega = L, f_i \in (-\gamma_i, \gamma_i), \exists j \neq i : s_j = s_L \mid m_k^i = H\right) \times \left(TP_L - s_M^i\right) \\
&+ \Pr(\omega = L, f_i \in (-\gamma_i, \gamma_i), \forall j \neq i : s_j = s_M^j \mid m_k^i = H\Big) \times \left(TP_{ML} - s_M^i\right).
\end{aligned}$$

Now, we use the Bayes rule to calculate $\Pr\left(\omega = H, f_i \in (-\gamma_i, \gamma_i), \exists j \neq i : s_j = s_H \mid m_k^i = H\right)$.

$$\begin{aligned}
\Pr(\ \omega = H, f_i \in (-\gamma_i, \gamma_i), \exists j \neq i : s_j = s_H \mid m_k^i = H) = \frac{1}{\Pr\left(m_k^i = H\right)} \times \Pr(\omega = H) \\
\times \Pr\left(f_i \in (-\gamma_i, \gamma_i) \mid \omega = H\right) \times \Pr\left(\exists j \neq i : s_j = s_H \mid \omega = H, f_i \in (-\gamma_i, \gamma_i)\right) \\
\times \Pr\left(m_k^i = H \mid \omega = H, f_i \in (-\gamma_i, \gamma_i), \exists j \neq i : s_j = s_H\right)\right) = \theta\gamma_i\left(1 - \beta_i\right)
\end{aligned}$$

We have used the following facts in the last equation, including:

$$\begin{aligned}
&\Pr\left(\exists j \neq i : s_j = s_H \mid \omega = H, f_i \in (-\gamma_i, \gamma_i)\right) = \Pr\left(\exists j \neq i : s_j = s_H \mid \omega = H\right) = 1 - \beta_i; \\
&\Pr(m_k^i = H \mid \omega = H, f_i \in (-\gamma_i, \gamma_i), \exists j \neq i : s_j = s_H = \Pr(m_k^i = H \mid \omega = H) = \theta; \\
&\Pr(m_k^i = H) = \sum_{\omega \in \{H,L\}} \Pr(\omega) \Pr(m_k^i = H \mid \omega) = \frac{1}{2}.
\end{aligned}$$

Similarly, we have:

$$\begin{aligned}
&\Pr\left(\omega = H, f_i \in (-\gamma_i, \gamma_i), \forall j \neq i : s_j = s_M^j \mid m_k^i = H\right) = \theta\gamma_i\beta_i; \\
&\Pr\left(\omega = L, f_i \in (-\gamma_i, \gamma_i), \exists j \neq i : s_j = s_L \mid m_k^i = H\right) = \gamma_i(1 - \theta)\left(1 - \beta_i\right); \\
&\Pr\left(\omega = L, f_i \in (-\gamma_i, \gamma_i), \forall j \neq i : s_j = s_M^j \mid m_k^i = H\right) = \gamma_i\beta_i(1 - \theta).
\end{aligned}$$

Plugging these conditional probabilities back into the formula of $\Pi_i(+1,H)$, we have:

$$\Pi_i(+1,H) = \frac{(2\theta - 1)\gamma_i(2 + \beta_i(n-1))\left((A_H - MC)^2 - (A_L - MC)^2\right)}{4(n+1)^2 b} > 0$$

If instead the speculator sells, his expected profit is

$$\Pi_i(-1,H) = -\frac{(2\theta-1)\gamma_i(2+\beta_i(n-1))\left((A_H-MC)^2-(A_L-MC)^2\right)}{4(n+1)^2 b} < 0$$

Thus, the optimal trading strategy is to buy $x_k^i = +1$ when $m_k^i = H$.

Symmetric reasoning shows that the speculator's optimal trading strategy is to sell $x_k^i = +1$ when $m_k^i = L$. And in this case, his trading profit satisfies $\Pi_i(-1,L) = \Pi_i(+1,H)$. Furthermore, since $(A_H-MC)^2-(A_L-MC)^2 = 2\left(\bar{A}-MC\right)(A_H-A_L)$, we conclude that

$$\Pi_i = \frac{(2\theta-1)\gamma_i\left(2+(n-1)\beta_i\right)\left(\bar{A}-MC\right)(A_H-A_L)}{2(n+1)^2 b}.$$

The proof concludes. □

A.3 Proof of Proposition 1

Proof. By Equation (8), $\frac{\partial\Pi(\alpha)}{\partial\alpha} < 0$. Thus, $\Pi(0) > \Pi(\alpha) > \Pi(1)$ for all $\alpha \in (0,1)$. Furthermore, by definition, we have: (i) when $c \geq \Pi(0) =: \bar{c}$, $\Pi(\alpha) < 0$ for any $\alpha > 0$, and thus $\widehat{\alpha} = 0$; (ii) when $c \leq \Pi(1) =: \underline{c}$, $\Pi(\alpha) < 0$ for any $\alpha < 1$, and thus $\widehat{\alpha} = 1$; and (iii) when $c \in (\underline{c}, \bar{c})$, by the intermediate value theorem and $\Pi(0) - c > 0 > \Pi(1) - c$, there exists a solution $\widehat{\alpha}$ such that $\Pi(\widehat{\alpha}) = c$, which is also unique since $\Pi'(\alpha) < 0$. □

A.4 Proof of Proposition 2

Proof. First, we can use Equation (8) to calculate the partial derivatives:

$$\begin{aligned}
\frac{\partial\Pi\left(n,\widehat{\alpha}_n\right)}{\partial\widehat{\alpha}_n} &= -\frac{(2\theta-1)^2\left(2+n(n-1)\widehat{\gamma}^{n-1}\right)\left(\bar{A}-MC\right)(A_H-A_L)}{2b(n+1)^2} \\
\frac{\partial\Pi\left(n,\widehat{\alpha}_n\right)}{\partial n} &= -\frac{\widehat{\gamma}_n(2\theta-1)\left(\bar{A}-MC\right)(A_H-A_L)}{2b(n+1)^3}\left\{4+\widehat{\gamma}_n^{n-1}\left(n-3+\left(n^2-1\right)\ln\frac{1}{\widehat{\gamma}_n}\right)\right\}
\end{aligned}$$

where $\widehat{\gamma}_n = 1-\widehat{\alpha}_n(2\theta-1)$.

By the implicit function theorem, we further have:

$$\begin{aligned}
\frac{\partial\widehat{\alpha}_n}{\partial n} &= -\left(\frac{\partial\Pi\left(n,\widehat{\alpha}_n\right)}{\partial n}\right)\Bigg/\left(\frac{\partial\Pi\left(n,\widehat{\alpha}_n\right)}{\partial\widehat{\alpha}_n}\right) \\
&= -\frac{\widehat{\gamma}_n{}^n}{(2\theta-1)(n+1)\left(2+n(n-1)\widehat{\gamma}_n{}^{n-1}\right)}\left(4\widehat{\gamma}_n^{1-n}+n-3+(n+1)(n-1)\ln\frac{1}{\widehat{\gamma}_n}\right) \qquad \text{(A.4)}
\end{aligned}$$

Obviously, when $n \geq 3$, it is easy to verify that $\frac{\partial\widehat{\alpha}_n}{\partial n} < 0$. Furthermore, we next show that $\frac{\partial\widehat{\alpha}_n}{\partial n} < 0$ holds when $n = 2$. Plugging in $n = 2$, it yields:

$$\frac{\partial\widehat{\alpha}_n}{\partial n}\Bigg| n = 2 = -\frac{\widehat{\gamma}_2^2}{6(2\theta-1)\left(1+\widehat{\gamma}_2\right)}\left(4\widehat{\gamma}_2^{-1}+3\ln\frac{1}{\widehat{\gamma}_2}-1\right)$$

Since $0 \leq \widehat{\gamma}_n = 1-\widehat{\alpha}_n(2\theta-1) \leq 1$, the result follows. The proof concludes. □

A.5 Proof of Corollary 1

Proof. We first show that $\frac{\partial \widehat{\alpha}_n}{\partial A_H} > 0$. Applying the implicit function theorem implies:

$$\frac{\partial \widehat{\alpha}_n}{\partial A_H} = -\left(\frac{\partial \Pi\left(\widehat{\alpha}_n\right)}{\partial A_H}\right) \Big/ \left(\frac{\partial \Pi\left(\widehat{\alpha}_n\right)}{\partial \widehat{\alpha}_n}\right)$$

We have already shown in the proof of Proposition 2 that $\frac{\partial \Pi(\widehat{\alpha}_n)}{\partial \widehat{\alpha}_n} < 0$. Hence, it suffices to show that $\frac{\partial \Pi(\widehat{\alpha}_n)}{\partial A_H} > 0$. Again, Using Equation (8), we obtain:

$$\frac{\partial \Pi\left(\widehat{\alpha}_n\right)}{\partial A_H} = \frac{2\widehat{\gamma}_n(2\theta-1)\left(A_H - MC\right)\left(2+(n-1)\widehat{\gamma}_n{}^{n-1}\right)}{4b(n+1)^2} > 0$$

Similarly, we can show that:

$$\begin{aligned}
\frac{\partial \Pi\left(\widehat{\alpha}_n\right)}{\partial A_L} &= -\frac{2\widehat{\gamma}_n(2\theta-1)\left(A_L - MC\right)\left(2+(n-1)\widehat{\gamma}_n{}^{n-1}\right)}{4b(n+1)^2} < 0,\\
\frac{\partial \Pi\left(\widehat{\alpha}_n\right)}{\partial MC} &= -\frac{\widehat{\gamma}_n(2\theta-1)\left(A_H - A_L\right)\left(2+(n-1)\widehat{\gamma}_n{}^{n-1}\right)}{2b(n+1)^2} < 0,\\
\frac{\partial \Pi\left(\widehat{\alpha}_n\right)}{\partial b} &= -\frac{\widehat{\gamma}_n(2\theta-1)\left(\bar{A} - MC\right)\left(A_H - A_L\right)\left(2+(n-1)\widehat{\gamma}_n{}^{n-1}\right)}{2b^2(n+1)^2} < 0.
\end{aligned}$$

Hence, $\frac{\partial \widehat{\alpha}_n}{\partial A_L} < 0$, $\frac{\partial \widehat{\alpha}_n}{\partial MC} < 0$, and $\frac{\partial \widehat{\alpha}_n}{\partial b} < 0$. The proof concludes. □

A.6 Derivation of Equation (14) and (15)

From Lemma 1 and Equation (12), we can calculate total welfare at $t = 1$ as

$$W = \begin{cases} W_H & \text{if } s_i = s_H \text{ for some } i \in \{1, \ldots, n\}; \\ W_{MH} & \text{if } \omega = H \ \& \ s_i = s_M^i \ \forall i \in \{1, \ldots, n\}; \\ W_{ML} & \text{if } \omega = L \ \& \ s_i = s_M^i \ \forall i \in \{1, \ldots, n\}; \text{and} \\ W_L & \text{if } s_i = s_L \text{ for some } i \in \{1, \ldots, n\}. \end{cases}$$

where $W_H = \frac{n(n+2)(A_H - MC)^2}{2b(n+1)^2}$, $W_{MH} = \frac{n\left(\bar{A}-MC\right)\left((2n+4)(A_H-MC)+n(A_H-A_L)\right)}{4b(n+1)^2}$, $W_L = \frac{n(n+2)(A_L-MC)^2}{2b(n+1)^2}$, and $W_{ML} = \frac{n\left(\bar{A}-MC\right)\left((2n+4)(A_L-MC)+n(A_L-A_H)\right)}{4b(n+1)^2}$.

Then, the expected total welfare is given by:

$$\begin{aligned}
\overline{W} &= \Pr\left(\exists i : s_i = s_H\right) \times W_H + \Pr\left(\forall i : s_i = s_M^i, \omega = H\right) \times W_{MH} \\
&+ \Pr\left(\exists i : s_i = s_L\right) \times W_L + \Pr\left(\forall i : s_i = s_M^i, \omega = L\right) \times W_{ML}
\end{aligned}$$

From the proof of Lemma 1, we already know that $f_i > \widehat{\gamma}_n$ (i.e., $s_i = s_H$) is impossible when $\omega = L$ and $f_i < \widehat{\gamma}_n$ (i.e., $s_i = s_L$) is impossible when $\omega = H$. Hence, we have:

$$\begin{aligned}
\overline{W} &= \Pr\left(\exists i : s_i = s_H, \omega = H\right) \times W_H + \Pr\left(\forall i : s_i = s_M^i, \omega = H\right) \times W_{MH} \\
&+ \Pr\left(\exists i : s_i = s_L, \omega = L\right) \times W_L + \Pr\left(\forall i : s_i = s_M^i, \omega = L\right) \times W_{ML}
\end{aligned}$$

To compute $\overline{W}$, we use the Bayes rule to calculate $\Pr(\exists i : s_i = s_H, \omega = H)$.

$$\Pr\left(\exists i : s_i = s_H, \omega = H\right) = \Pr(\omega = H)\Pr\left(\exists i : s_i = s_H \mid \omega = H\right)$$

Using the expression of $s_i(f_i)$ in Equation (6), we know:

$$\begin{aligned}\Pr\left(s_i = s_M^i \mid \omega = H\right) &= \Pr\left(-\widehat{\gamma}_n \leq f_i \leq \widehat{\gamma}_n \mid \omega = H\right) = \widehat{\gamma}_n \\ \Pr\left(s_i = s_H \mid \omega = H\right) &= \Pr\left(f_i > \widehat{\gamma}_n \mid \omega = H\right) = 1 - \widehat{\gamma}_n\end{aligned}$$

and thus: $\Pr\left(\exists i : s_i = s_H \mid \omega = H\right) = 1 - \Pr\left(\forall i : s_i = s_M^i \mid \omega = H\right) = 1 - (\widehat{\gamma}_n)^n$.

Since $\Pr(\omega = H) = 1/2$, we further have:

$$\Pr\left(\exists i : s_i = s_H, \omega = H\right) = \frac{1 - (\widehat{\gamma}_n)^n}{2}$$

Similarly, we have

$$\begin{aligned}&\Pr\left(\exists i : s_i = s_L, \omega = L\right) = \frac{1 - (\widehat{\gamma}_n)^n}{2}, \\ &\Pr\left(\forall i : s_i = s_M^i, \omega = H\right) = \Pr\left(\forall i : s_i = s_M^i, \omega = L\right) = \frac{(\widehat{\gamma}_n)^n}{2}\end{aligned}$$

Therefore, $\overline{W}$ can be written as

$$\overline{W}(\widehat{\alpha}_n, n) = \frac{n(n+2)}{8(n+1)^2 b}\left(4\left(\bar{A} - MC\right)^2 + \left(1 - (\widehat{\gamma}_n)^n\right)\left(A_H - A_L\right)^2\right)$$

Obviously, $\overline{W}$ depends on n and $\widehat{\alpha}_n$, which implicitly depends on n, and we can explicitly write: $\overline{W}(\widehat{\alpha}_n, n)$. Given the monotone relationship between $\widehat{\alpha}_n$ and n, we know that the expected total welfare is uniquely determined for any fixed n.

Last, note that we can show for the formula of $\overline{CS}(\widehat{\alpha}_n, n)$ in a similar way. Again, from Lemma 1 and Equation (12), we can calculate consumer surplus at $t = 1$ as

$$CS = \begin{cases} CS_H & \text{if } s_i = s_H \text{ for some } i \in \{1, \dots, n\}; \\ CS_{MH} & \text{if } \omega = H \ \& \ s_i = s_M^i \ \forall i \in \{1, \dots, n\}; \\ CS_{ML} & \text{if } \omega = L \ \& \ s_i = s_M^i \ \forall i \in \{1, \dots, n\}; \text{and} \\ CS_L & \text{if } s_i = s_L \text{ for some } i \in \{1, \dots, n\}. \end{cases}$$

where $CS_H = \frac{n^2(A_H - MC)^2}{2b(n+1)^2}$, $CS_L = \frac{n^2(A_L - MC)^2}{2b(n+1)^2}$, and $CS_{MH} = CS_{ML} = \frac{n^2\left(\bar{A} - MC\right)^2}{2b(n+1)^2}$

Furthermore, similar to $\bar{W}$, we have:

$$\begin{aligned}\overline{CS} = &\Pr\left(\exists i : s_i = s_H, \omega = H\right) \times CS_H + \Pr\left(\forall i : s_i = s_M^i, \omega = H\right) \times CS_{MH} \\ &+ \Pr\left(\exists i : s_i = s_L, \omega = L\right) \times CS_L + \Pr\left(\forall i : s_i = s_M^i, \omega = L\right) \times CS_{ML}\end{aligned}$$

Thus, $\overline{CS}$ can be calculated as

$$\overline{CS} = \frac{1 - (\widehat{\gamma}_n)^n}{2} \times (CS_H + CS_L) + \frac{(\widehat{\gamma}_n)^n}{2} \times (CS_{MH} + CS_{ML})$$

From the expression of the consumer surplus at $t=1$, we further have:

$$\overline{CS}\left(\widehat{\alpha}_n, n\right)=\frac{n^2}{8b(n+1)^2}\left(4\left(\bar{A}-MC\right)^2+\left(1-\left(\widehat{\gamma}_n\right)^n\right)\left(A_H-A_L\right)^2\right).$$

The derivation concludes.

A.7 Proof of Lemma 3

Proof. **(i) Total welfare.** Based on the expression for $\overline{W}(\widehat{\alpha}_n, n)$ in Equation (14), we know that

$$\frac{d\overline{W}\left(\widehat{\alpha}_n, n\right)}{dn}=\frac{\partial\overline{W}\left(\widehat{\alpha}_n, n\right)}{\partial n}+\frac{\partial\overline{W}\left(\widehat{\alpha}_n, n\right)}{\partial\widehat{\alpha}_n}\frac{\partial\widehat{\alpha}_n}{\partial n}$$

First, the partial derivative of $\overline{W}\left(\widehat{\alpha}_n, n\right)$ with respect to n can be calculated as

$$\begin{aligned}\frac{\partial\overline{W}\left(\widehat{\alpha}_n, n\right)}{\partial n} &= \frac{n(n+2)\left(A_H-A_L\right)^2\left(\widehat{\gamma}_n\right)^n\ln(1/\widehat{\gamma}_n)}{8b(n+1)^2}\\ &+ \frac{1}{4b(n+1)^3}\left(4\left(\bar{A}-MC\right)^2+\left(1-\left(\widehat{\gamma}_n\right)^n\right)\left(A_H-A_L\right)^2\right)\end{aligned}$$

Second, we calculate the partial derivative of $\overline{W}\left(\widehat{\alpha}_n, n\right)$ with respect to $\widehat{\alpha}_n$ as follows:

$$\frac{\partial\overline{W}\left(\widehat{\alpha}_n, n\right)}{\partial\widehat{\alpha}_n}=\frac{\left(\widehat{\gamma}_n\right)^{n-1}n^2(n+2)(2\theta-1)\left(A_H-A_L\right)^2}{8b(n+1)^2}.$$

Using Equations (A.4) and the two partial derivatives above, we get:

$$\frac{d\overline{W}\left(\widehat{\alpha}_n, n\right)}{dn}=\frac{\left(A_H-A_L\right)^2}{8b(n+1)^3}\left\{\frac{2\left(4\left(\bar{A}-MC\right)^2+\left(A_H-A_L\right)^2\right)}{\left(A_H-A_L\right)^2}-g_1\left(\widehat{\alpha}_n, n\right)\right\}$$

Therefore, $\frac{d\overline{W}(\widehat{\alpha}_n, n)}{dn}<0$ holds if and only if: $g_1\left(\widehat{\alpha}_n, n\right)>\frac{8\left(\bar{A}-MC\right)^2}{\left(A_H-A_L\right)^2}+2$.

(ii) Consumer surplus. Obviously, $\overline{CS}\left(\widehat{\alpha}_n, n\right)=\frac{n}{n+2}\overline{W}\left(\widehat{\alpha}_n, n\right)$. Thus, the total derivative of $\overline{CS}\left(\widehat{\alpha}_n, n\right)$ with respect to n can be written as follows:

$$\frac{d\overline{CS}\left(\widehat{\alpha}_n, n\right)}{dn}=\frac{n}{n+2}\times\frac{d\overline{W}\left(\widehat{\alpha}_n, n\right)}{dn}+\frac{2}{(n+2)^2}\times\overline{W}\left(\widehat{\alpha}_n, n\right)$$

Recall that $\overline{W}\left(\widehat{\alpha}_n, n\right)=\frac{n(n+2)}{8b(n+1)^2}\left\{4\left(\bar{A}-MC\right)^2+\left(1-\left(\widehat{\gamma}_n\right)^n\right)\left(A_H-A_L\right)^2\right\}$ and $\frac{d\overline{W}(\widehat{\alpha}_n, n)}{dn}=\frac{\left(A_H-A_L\right)^2}{8b(n+1)^3}\left(G_1-g_1\left(\widehat{\alpha}_n, n\right)\right)$. Then, we can calculate $d\overline{CS}\left(\widehat{\alpha}_n, n\right)/dn$ as follows:

$$\frac{d\overline{CS}\left(\widehat{\alpha}_n, n\right)}{dn}=\frac{n\left(A_H-A_L\right)^2}{8b(n+1)^3}\left(G_1-g_2\left(\widehat{\alpha}_n, n\right)\right)$$

Thus, $\frac{d\overline{CS}(\widehat{\alpha}_n, n)}{dn}<0$ holds if and only if $g_2\left(\widehat{\alpha}_n, n\right)>G_1$ is true. The proof concludes. □

A.8 Proof of Proposition 3

Proof. The idea is to construct a set U of the information production cost such that for any $c \in U$, we have: (i) $\hat{\alpha}_m = 1$, $\hat{\alpha}_n = 0$; (ii) $n > m$; and (iii) $\overline{W}(\widehat{\alpha}_m, m) > \overline{W}(\widehat{\alpha}_n, n)$. It suffices to show that competition can decrease total welfare through informational feedback when $U \neq \varnothing$, because whenever information production is fixed, an increase in the number of firms always improves total welfare in Cournot competition.

Now, we come to construct U. First, given condition (i),

$$\frac{\overline{W}(\widehat{\alpha}_m, m)}{\overline{W}(\widehat{\alpha}_n, n)} = \frac{\left(1 - \frac{1}{(m+1)^2}\right) * (1 + \mu * (1 - (2 - 2\theta)^m))}{\left(1 - \frac{1}{(n+1)^2}\right)}$$

Thus, $\overline{W}(\widehat{\alpha}_m, m) > \overline{W}(\widehat{\alpha}_n, n)$ holds whenever $\Phi(m) \geq 1$ is true, since the denominator is always smaller than m for any $n \in \mathbb{N}$.

Second, since $\Phi(m)$ is continuous and strictly increasing in m and that $\lim_{l \to \infty} \Phi(m) = (1+\mu) > 1$, there exists some m_0 sufficiently large such that $\Phi(m) \geq 1$ for all $m \geq m_0$. Fix any m such that $\Phi(m) \geq 1$, and we can define $\underline{c}_m$ by Equation (10).

Third, we can use the floor function $[x] = \{z \in \mathbb{Z} : z \leq x\}$ to define:

$$N(m) = \frac{(m+1)^2}{(2 - 2\theta)(2 + (m-1)(2 - 2\theta)^{m-1})}$$

By construction, we have $\underline{c}_m > \bar{c}_N$. Therefore, we can define $U = [\bar{c}_n, \underline{c}_m)$ for any $n \geq N$ because $\bar{c}_n$ is strictly decreasing in n. By construction, $U = [\bar{c}_n, \underline{c}_m)$ is the desired set that satifies conditions (i)-(iii). The proof concludes. □

A.9 Proof of Proposition 4

Proof. We prove this result for all parameters one by one.

Case (i): Information production cost c**.** First, when $c = 0$, $\hat{\alpha}_n = 1$ for all $n \in \mathbb{N}$. Therefore, $n^* \to \infty$. Second, when $c > \bar{c}_1$, then $\hat{\alpha}_n = 0$, and thus $n^* \to \infty$. Then, the non-monotonicity of $n^*(c)$ follows from Corollary A.1 below.

Corollary A.1. *Consider n_1 such that $\Phi(n_1) \geq 1$ and $n_2 \geq N(n_1)$. Then:*

(1) When $c < \underline{c}_{n_2}$ or $c > \bar{c}_{n_1}$, $\overline{W}(\widehat{\alpha}_{n_2}, n_2) > \overline{W}(\widehat{\alpha}_{n_1}, n_1)$; and

(2) When $\bar{c}_{n_2} < c < \underline{c}_{n_1}$, $\overline{W}(\widehat{\alpha}_{n_2}, n_2) < \overline{W}(\widehat{\alpha}_{n_1}, n_1)$.

Note that Corollary A.1 follows directly from Proposition 3.

Case (ii): Price sensitivity b**.** First, when $b \to \infty$, we have $\Pi(\alpha) \to 0$, which implies that $\hat{\alpha}_n = 0$ for all $n \in \mathbb{N}$ and thus $n^* \to \infty$. Second, when $b \to 0$, then $\hat{\alpha}_n = 1$, and thus $n^* \to \infty$. Then, the non-monotonicity of $n^*(b)$ follows from Corollary A.1. To see it, select positive integers n_1 and n_2 such that: $\Phi(n_1) \geq 1$ and $n_2 \geq N(n_1)$. By Corollary A.1, $n^* < n_2$ when $\bar{c}_{n_2} < c < \underline{c}_{n_1}$, which translates into:

$$\frac{(2\theta - 1)(A_H - A_L)(\bar{A} - MC)}{2(n_2 + 1)c} < b < \frac{(2\theta - 1)(1 - \theta)(2 + (n_1 - 1)(2 - 2\theta)^{n_1 - 1})(A_H - A_L)(\bar{A} - MC)}{(n_1 + 1)^2 c}$$

Therefore, n^* is non-monotonic in b.

Case (iii): Market prospect in good state A_H. First, when $A_H \to \infty$, we have $\Pi(\alpha) \to \infty$, which implies that $\hat{\alpha}_n = 1$ for all $n \in \mathbb{N}$ and thus $n^* \to \infty$. Second, when $(A_H - A_L) \to 0$, then $\hat{\alpha}_n = 0$, and thus $n^* \to \infty$. Then, the non-monotonicity of n^* follows from Corollary A.1. To see it, select positive integers n_1 and n_2 such that: $\Phi(n_1) \geq 1$ and $n_2 \geq N(n_1)$. By Corollary A.1, $n^* < n_2$ when $\bar{c}_{n_2} < c < \underline{c}_{n_1}$, which translates into:

$$A_L + \frac{2(n_2+1)bc}{(2\theta-1)(\bar{A}-MC)} > A_H > A_L + \frac{(n_1+1)^2 bc}{(2\theta-1)(1-\theta)(2+(n_1-1)(2-2\theta)^{n_1-1})(\bar{A}-MC)}$$

Thus, $n^* < \infty$ can be finite. Therefore, n^* is non-monotonic in $(A_H - A_L)$. The proof concludes. $\square$

A.10 Proof of Lemma 4

Proof. First, note that by the assumed condition $A_L = MC$, $4(\bar{A} - MC)^2 = (A_H - A_L)^2$. Thus, $\overline{W}(\widehat{\alpha}_1, 1) > \overline{W}(\widehat{\alpha}_2, 2)$ reduces to:

$$\frac{3}{32}(2-\widehat{\gamma}_1) > \frac{1}{9}(2-(\widehat{\gamma}_2)^2)$$

Second, when $c \geq \frac{(2\theta-1)(A_H-A_L)^2}{12b}$, by Equation (9), we have: $\widehat{\alpha}_2 = 0$ and thus $\widehat{\gamma}_2 = 1$. This further implies that $\overline{W}(\widehat{\alpha}_1, 1) > \overline{W}(\widehat{\alpha}_2, 2)$ if and only if $\widehat{\gamma}_1 < \frac{22}{27}$.

Finally, note that $\widehat{\gamma}_1$ is governed by Equation (8). Simple algebra yields the bound $c \leq \frac{11}{108}\kappa$. The other condition $c < \frac{(1-\theta)(2-\theta)\kappa}{9}$ follows from the definition of $\underline{c}$ for $n = 1$ and $n = 2$. Indeed, if $c < \min\{\underline{c}_1, \underline{c}_2\}$, then $\widehat{\gamma}_1 = \widehat{\gamma}_2 = 1$, and thus $\overline{W}(\widehat{\alpha}_1, 1) \leq \overline{W}(\widehat{\alpha}_2, 2)$. The proof concludes. $\square$

Online Appendix

B Extended Discussions

B.1 Impacts of feedback effects from stock market: an alterative scenario

Under extreme parameter values, where low market uncertainty reduces the informational value of managerial learning, the stock market feedback effect may not overturn the positive relationship between competition and total welfare. Nonetheless, it can significantly shape the efficiency implications of firm competition, making it a crucial factor in regulating horizontal mergers.

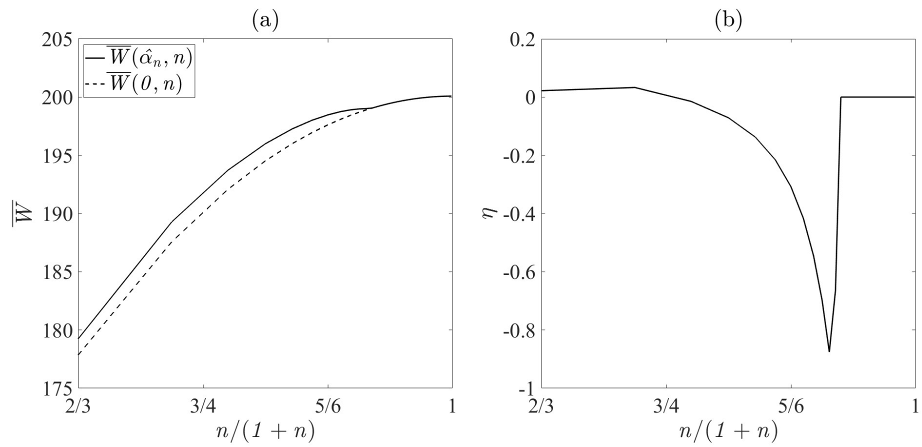


Figure 11: Small Market Uncertainty ($A_L = 25$)

Notes: This figure estimates the total welfare with and without feedback effects, as well as $\eta = \frac{\overline{W}(\widehat{\alpha}_n,n)-\overline{W}(\widehat{\alpha}_{n-1},n-1)}{\overline{W}(0,n)-\overline{W}(0,n-1)} - 1$. A negative value of η indicates that the welfare effect of a horizontal merger will be overestimated if the feedback effect is ignored. A positive value of η then suggests that the feedback effect augments the welfare effect of a horizontal merger.

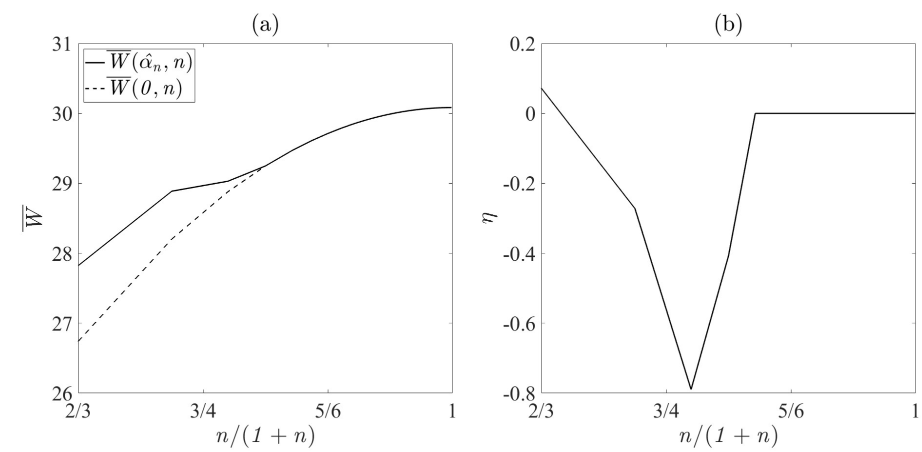
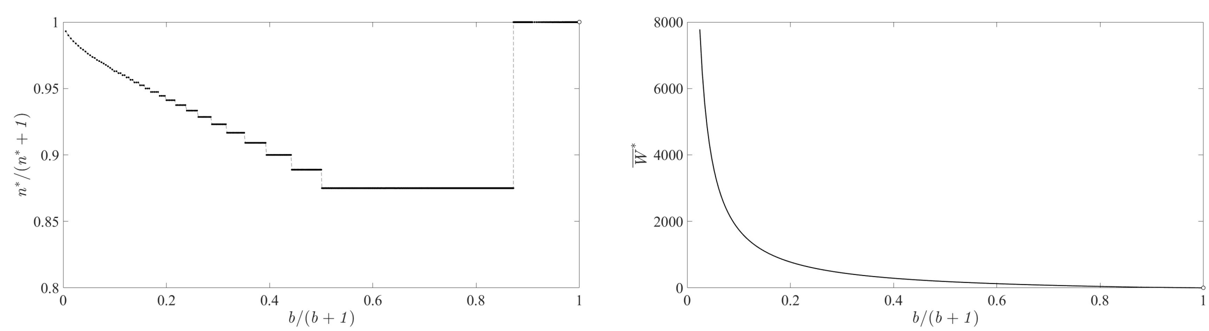


Figure 12: Small Market Uncertainty ($A_H = 15$)

Compared to the baseline model, Figures 11 and 12 adjust the parameter values of A_L from 10 to 25 and A_H from 30 to 15, respectively, while keeping all other parameters unchanged. These modifications are quite extreme, reducing the ratio $\frac{A_H - A_L}{MC}$ by 75%, from $\frac{20}{3}$ to $\frac{5}{3}$. Under these two sets of parameter configurations, the feedback effects are insufficient to reverse the positive relationship between firm competition and total welfare. Nevertheless, the feedback effect continues to exert a significant influence on the efficiency implications of competition. Specifically, when the intensity of firm competition varies, the welfare change without considering feedback effects can be substantially smaller — by as much as 80%.

B.2 An Extended Discussion for Section 4.4

Price sensitivity b**.** Figure 13 depicts the optimal market structure $n^*/(n^*+1)$ and the corresponding total welfare $W(n^*)$ under the optimal market structure n^*. When b is high, the market price is very sensitive to the quantity of production, reducing profits for the firms and thus discouraging the production of information. Therefore, the information production gap disappears when we vary n, leading to a dominant role of market power concentration. Similarly, when b is low, the market price is insensitive, increasing profits for all firms and thus enhancing information production. Again, the information production gap disappears when we vary n, and the market concentration channel becomes dominant. For an intermediate level of price sensitivity b, the information production gap can be relatively large when changing the number of firms in the market, and the information production channel can dominate that of market concentration. This pattern is illustrated in Figure 13a. However, note that a decrease in b always improves total welfare, because it directly increases firms' profits and consumer welfare and indirectly improves total welfare by enhancing information production.

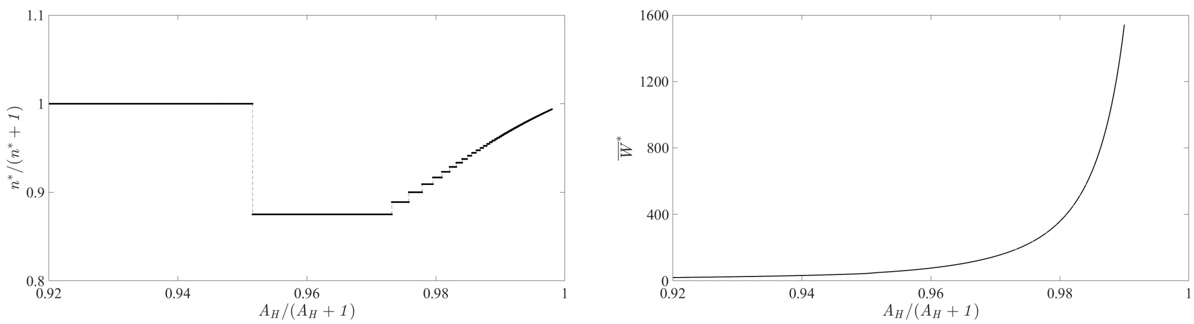


(a) Optimal Market Structure n^* (b) Total Welfare $W(n^*)$

Figure 13: Price Sensitivity b

Parameters: $\theta = 0.75$, $c = 1.5$, $MC = 3$, $A_H = 30$, $A_L = 10$.

Market prospect parameters A_H**.** Figure 14 depicts n^* and $W(n^*)$ when we vary the market prospect A_H in the good state $\omega = H$. Specifically, when A_H increases from zero to ∞, the optimal market structure n^* first decreases and then increases. Similar to other parameters, the total welfare under the optimal market structure always increases in A_H. Unlike other parameters, A_H affects the equilibrium through two forces, including market uncertainty $(A_H - A_L)$ and average profitability. These two forces can both increase information production (see, e.g., Equation (8)). However, their impacts on the optimal market structure can diverge, as illustrated in the discussion below, i.e., the negative relationship between competition and total welfare is more likely to occur when average profitability is relatively small (but not too tiny, otherwise the information production gap disappears) or the uncertainty is relatively large (but not too large). In other words, an increase in average profitability weakens, while an increase in market uncertainty reinforces the importance of the information production channel in the negative relationship between competition and total welfare.



(a) Optimal Market Structure n^* (b) Total Welfare $W(n^*)$

Figure 14: Market Prospect Paramter A_H

Parameters: $\theta = 0.75$, $b = 1.5$, $c = 1.5$, $MC = 3$, $A_L = 10$.

B.3 Calibration Based on US market data

This section provides a detailed explanation of the process used to estimate model parameters based on US market data. Before introducing the specific estimation procedure, we first clarify the parameters required to compute the impact of feedback effects, denoted as η.

Substituting the expression for $\overline{W}(\widehat{\alpha}_n, n)$ into Equation (19) and simplifying, we obtain:

$$\eta = \frac{T_W(\widehat{\alpha}_n, n) - T_W(\widehat{\alpha}_{n-1}, n-1)}{T_W(0,n) - T_W(0,n-1)} - 1 \tag{B.1}$$

where

$$T_W(\widehat{\alpha}_n, n) = \frac{n(n+2)}{8(n+1)^2}\left(\left(\frac{A_H}{MC} + \frac{A_L}{MC} - 2\right)^2 + (1-\hat{\gamma}_n^n)\left(\frac{A_H}{MC} - \frac{A_L}{MC}\right)^2\right).$$

Furthermore, using the equilibrium condition $\Pi(\widehat{\alpha}_n) = c$, we derive:

$$\frac{\gamma_n(2\theta-1)\left(2+(n-1)\gamma_n^{n-1}\right)\left(\frac{A_H}{MC} + \frac{A_L}{MC} - 2\right)\left(\frac{A_H}{MC} - \frac{A_L}{MC}\right)}{4(n+1)^2} = \frac{b*c}{MC^2}. \tag{B.2}$$

From Equations (B.1) and (B.2), we need to estimate the parameters n and θ, as well as the three ratios $\frac{A_H}{MC}$, $\frac{A_L}{MC}$, and $\frac{bc}{MC^2}$, to compute η. Without loss of generality, we assume $b = MC = 1$.[21] Additionally, since the information precision parameter θ is difficult to estimate from real-world data, we rely on the restriction $\theta \in (0.5, 1)$ and a reasonable compromise is to set $\theta = 0.75$.

Next, we proceed with estimating the remaining four parameters: n, A_H, A_L, and c. Specifically, we used US industry data to illustrate the parameter estimation process, which is similar for industry-specific estimations. The required data includes firm financial data from *Compustat* (1950–2023), analyst forecasts from *Zacks Investment Research Database* (2000–2023), and PIN data from *Stephen Brown's website* (1993–2010). The sample period for parameter estimation is 2000–2010. Following Gu (2016) and Hou and Robinson (2006), industries are classified using three-digit SIC codes from *CRSP*. Financial and utility firms, as well as industries with negative gross margins, are excluded to align with the Cournot model. Continuous variables are winsorized at the 1st and 99th percentiles to reduce extreme value effects.

First, we estimate competition intensity n using the **Herfindahl-Hirschman Index (HHI)**. Following Gu (2016), we can define:

$$HHI_{jt} = \sum_{i=1}^{N_j} s_{ijt}^2,$$

where s_{ij} is firm i's market share in industry j in year t, and N_j is the number of firms. Market share is computed as *net sales* (Compustat *SALE*) divided by total industry sales. The sample mean of US industry HHI is 0.361. In the Cournot model, with n homogeneous firms, $HHI = \sum_{i=1}^{n} \frac{1}{n^2} = \frac{1}{n}$. Thus, we estimate: $n = \frac{1}{0.361} \approx 3$.

Second, we will estimate A_H and A_L. Since these parameters are not directly convenient to estimate, we instead estimate the average profitability $\bar{A} - MC$ and market uncertainty $A_H - A_L$. First, we use the gross margin GM_{it} to estimate the average profitability $\bar{A} - MC$. The gross margin GM_{it} for each firm i in year t is calculated as one minus the cost of goods sold scaled by sales. From this, the sample mean of the gross margin for U.S. firms is calculated to be 0.236. In the Cournot model, the average gross margin (GM) can be expressed as:

$$GM = \frac{\bar{P} - MC}{\bar{P}} = \frac{\bar{A} - bnq_M - MC}{\bar{A} - bnq_M} = \frac{\bar{A} - MC}{\bar{A} + nMC}.$$

Using this, along with $MC = 1$ and $n = 3$, we can estimate $\bar{A} - MC = 1.236$.

Third, we estimate market uncertainty $A_H - A_L$ using analyst forecast errors, as they reflect both public market information and managerial insights, with higher uncertainty leading to larger errors. The mean absolute percentage error (MAPE) is calculated as:

$$\text{MAPE} = \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} \left| \frac{\text{Sales}_{\text{Fit}} - \text{Sales}_{\text{Ait}}}{\text{Sales}_{\text{Ait}}} \right| \times 100\%,$$

where i is the firm index, t is the year index, N is the number of firms, T is the number of years, $\text{Sales}_{\text{Ait}}$ is actual sales in year t, and $\text{Sales}_{\text{Fit}}$ is the median analyst forecast for year t in year

[21] Note that in Equation (B.2), the ratio $\frac{b*c}{MC^2}$, rather than b alone, enters the equilibrium condition and is related to the probability of misallocation in equilibrium. In calibration, we directly estimate the size of informed speculators α and the probability of misallocation γ.

$t-1$ (Polk and Sapienza, 2008). The MAPE is 0.292. Since MAPE measures relative market uncertainty, we compare it to the Coefficient of Variation (CV) of A:

$$CV = \frac{\sqrt{\Pr(\omega = H) \times \left(A_H - \bar{A}\right)^2 + \Pr(\omega = L) \times \left(A_L - \bar{A}\right)^2}}{\bar{A}} = \frac{A_H - A_L}{2\bar{A}}.$$

Given $\bar{A} - MC = 1.236$, we estimate $A_H - A_L = 1.306$, yielding $A_H = 2.889$ and $A_L = 1.583$.

Fourth, we estimate the information cost c using sample data of PIN (Probability of Informed Trading, see Easley et al. (1996)). Since PIN directly estimates the probability of informed trading (Easley et al., 1996), its sample mean provides a reasonable estimate of $\widehat{\alpha}$ at equilibrium, allowing us to estimate c. With a full-sample mean of PIN equal to 0.233, we substitute $\widehat{\alpha} = 0.233$ and the other estimated parameters into equation (B.2), yielding $c = 0.079$. A similar approach allows for parameter estimation across industries.

In addition, we use parameters calibrated from US market data to redraw Figures 3-8.

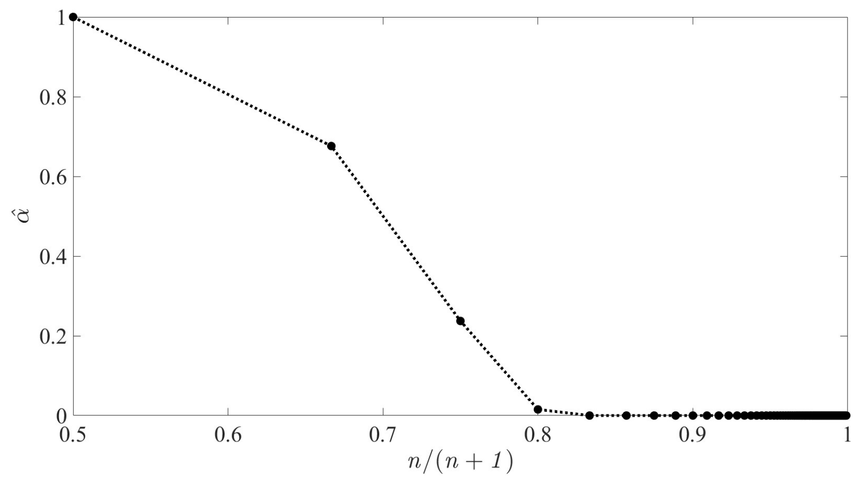


Figure 15: Production Competition and Information Production (Calibrated Data)

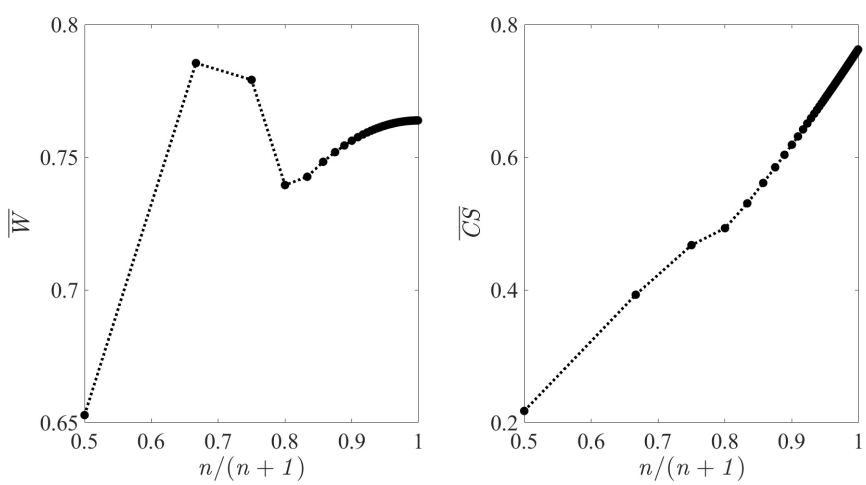


Figure 16: Competition, Total Welfare and Consumer Welfare (Calibrated Data)

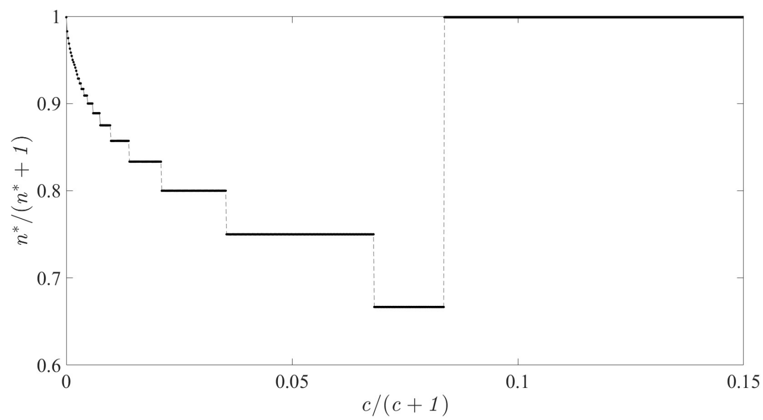


Figure 17: Optimal Market Structure (Calibrated Data)

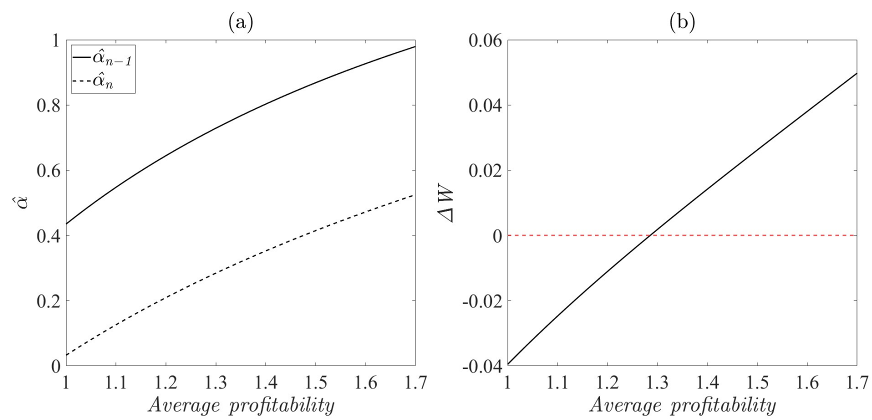


Figure 18: Average Profitability (Calibrated Data)

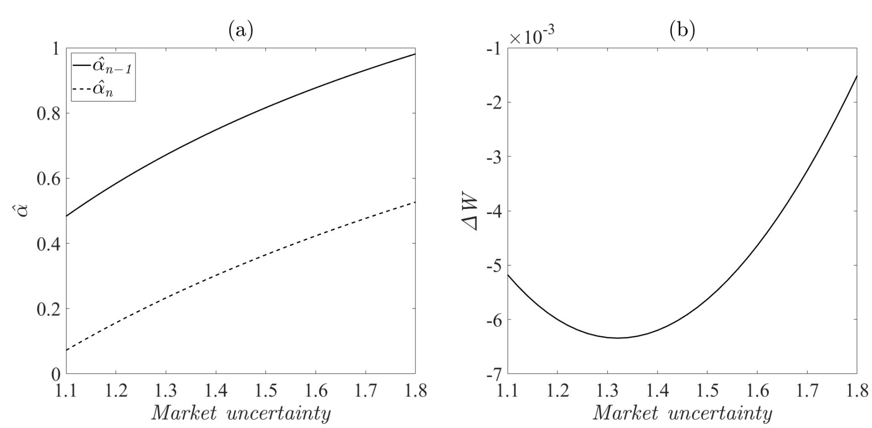


Figure 19: Market Uncertainty (Calibrated Data)

B.4 Equilibrium Analysis in Section 5.1

This section analyzes the equilibrium for the cross-asset trading setup in Section 5.1. We first solve the equilibrium, taking as given the measures of informed speculators $\boldsymbol{\alpha}$, which is then determined by investigating the incentive for information acquisition. Analogous to Lemma 1, given $\boldsymbol{\alpha}$, the stock price $s_i(f_i)$ is determined as:

$$s_i\left(f_i\right)=\begin{cases} s_H & \text{if } \ f_i\in(\gamma_i^{LS},\infty);\\ s_M^i & \text{if } \ f_i\in[-\gamma_i^{LS},\gamma_i^{LS}];\\ s_L & \text{if } \ f_i\in(-\infty,-\gamma_i^{LS}).\end{cases}$$

where $s_H=\frac{(A_H-MC)^2}{(n+1)^2b}$, $s_M^i=\frac{1}{4(n+1)^2b}\left(2\left(A_H-MC\right)^2+2\left(A_L-MC\right)^2-\beta_i^{LS}\left(A_H-A_L\right)^2\right)$, $s_L=\frac{(A_L-MC)^2}{(n+1)^2b}$, $\gamma_i^{LS}=1-(2\theta-1)(\alpha_L+\alpha_{i,S})$ and $\beta_i^{LS}=\prod_{j\neq i}\gamma_i^{LS}$.

Furthermore, the ith firm's optimal production strategy, conditional on the stock prices observed, is given by:

$$q_i^*(\boldsymbol{s})=\begin{cases} q_H & \text{if } \ \exists j\in\{1,\ldots,n\}:s_j=s_H;\\ q_M & \text{if } \ \forall j\in\{1,\ldots,n\}:s_j=s_M^j;\\ q_L & \text{if } \ \exists j\in\{1,\ldots,n\}:s_j=s_L.\end{cases}$$

where $q_H=\frac{A_H-MC}{(n+1)b}$, $\bar{A}=\frac{1}{2}\left(A_H+A_L\right)$, $q_M=\frac{\bar{A}-MC}{(n+1)b}$, and $q_L=\frac{A_L-MC}{(n+1)b}$.

Next, we endogenize the measure of informed traders $\boldsymbol{\alpha}$. Specifically, for an informed L-trader k with a private signal m_k, the optimal trading strategy is to hold $y_k^j=+1$ ($y_k^j=-1$) share of each firm $j\in\{1,\ldots,n\}$ when $m_k=H$ ($m_k=L$), leading to an expected trading profit given by:

$$\Pi_L(\boldsymbol{\alpha})=\frac{\left(\bar{A}-MC\right)\left(A_H-A_L\right)(2\theta-1)\sum_{j=1}^{n}\gamma_j^{LS}\left(2+(n-1)\beta_j^{LS}\right)}{2b(n+1)^2}$$

Similarly, for an informed S-trader k with a private signal m_k^i, the optimal trading strategy is to buy $x_k^i=+1$ shares of the ith stock when $m_k^i=H$, and sell $x_k^i=-1$ shares of the ith stock when $m_k^i=L$. This leads to an expected trading profit:

$$\Pi_S^i(\boldsymbol{\alpha})=\frac{\left(\bar{A}-MC\right)\left(A_H-A_L\right)(2\theta-1)\gamma_i^{LS}\left(2+(n-1)\beta_i^{LS}\right)}{2b(n+1)^2}$$

Since all firms in the Cournot competition are identical, we can focus on the symmetric equilibrium in which $\alpha_{i,S}=\alpha_S$. Then, with information acquisition, the expected profits for the L- and S-traders can be further written as: $\Pi_L(\boldsymbol{\alpha})=n\Pi_S(\boldsymbol{\alpha})$ and

$$\Pi_S(\boldsymbol{\alpha})=\Pi_S(\alpha_L,\alpha_S)=\frac{\left(\bar{A}-MC\right)\left(A_H-A_L\right)(2\theta-1)\gamma^{LS}\left(2+(n-1)(\gamma^{LS})^{n-1}\right)}{2b(n+1)^2}\tag{B.3}$$

where $\gamma^{LS}=1-(2\theta-1)(\alpha_L+\alpha_S)$.

By comparing $\Pi_L(\boldsymbol{\alpha})$ and $\Pi_S(\boldsymbol{\alpha})$, we can observe that L-traders have a stronger incentive to acquire information than S-traders, given that $c_L\leq c_S$.This further implies: (1) if $\alpha_S>0$, then

$\alpha_L = \lambda$; and (2) if $\alpha_L < \lambda$, then $\alpha_S = 0$. Using this property, we can derive the optimal strategies for information production as follows.

Lemma B.1 (Information Production)**.** *The equilibrium intensity of information production* $(\widetilde{\alpha}_L, \widetilde{\alpha}_S)$ *satisfies the following:*

(i) when $c_L \geq \Pi_L(0,0)$*, then* $\widetilde{\alpha}_L = \widetilde{\alpha}_S = 0$*;*

(ii) when $\Pi_L(\lambda,0) < c_L < \Pi_L(0,0)$*, then* $\widetilde{\alpha}_S = 0$ *and* $\widetilde{\alpha}_L \in (0,\lambda)$*, where* $\Pi_L(\widetilde{\alpha}_L, 0) = c_L$*;*

(iii) when $c_L < \Pi_L(\lambda,0)$ *and* $c_S \geq \Pi_S(\lambda,0)$*, then* $\widetilde{\alpha}_L = \lambda$ *and* $\widetilde{\alpha}_S = 0$*;*

(iv) when $c_L < \Pi_L(\lambda,0)$ *and* $\Pi_S(\lambda, 1-\lambda) < c_S < \Pi_S(\lambda,0)$*, then* $\widetilde{\alpha}_L = \lambda$ *and* $\widetilde{\alpha}_S \in (0, 1-\lambda)$*, where* $\Pi_S(\lambda, \widetilde{\alpha}_S) = c_S$*; and*

(v) when $c_L < \Pi_L(\lambda,0)$ *and* $c_S \leq \Pi_S(\lambda, 1-\lambda)$*, then* $\widetilde{\alpha}_L = \lambda$ *and* $\widetilde{\alpha}_S = 1-\lambda$*.*

Define $\widetilde{\boldsymbol{\alpha}}_n := \widetilde{\boldsymbol{\alpha}}(n)$. Finally, following the derivation of Equation (14), we can compute the expected total welfare $\widetilde{W}(\widetilde{\boldsymbol{\alpha}}_n, n)$ as follows:

$$\widetilde{W}(\widetilde{\boldsymbol{\alpha}}_n, n) = \frac{n(n+2)}{8b(n+1)^2}\left(4\left(\bar{A} - MC\right)^2 + \left(1 - (\widetilde{\gamma}^{LS})^n\right)(A_H - A_L)^2\right) \tag{B.4}$$

where $\widetilde{\gamma}^{LS} = 1 - (\widetilde{\alpha}_L + \widetilde{\alpha}_S) \times (2\theta - 1)$.

Furthermore, define $\gamma_S = 1 - (2\theta-1)(\lambda + \widetilde{\alpha}_S)$, $\gamma_L = 1 - \widetilde{\alpha}_L(2\theta - 1)$,

$$g_S(\widetilde{\alpha}_S, n) = 2\gamma_S^n + \frac{n(n+2)\gamma_S^n}{2 + n(n-1)\gamma_S^{n-1}}\left(4n + n(n-3)\gamma_S^{n-1} - 2(n+1)\ln\frac{1}{\gamma_S}\right)$$

and

$$g_L(\widetilde{\alpha}_L, n) = \frac{(\gamma_L)^n \times \left(2n(n-1)(n+2) + 4 - 3n^2(n+1)\gamma_L^{n-1} - 2n(n+1)(n+2)\ln\frac{1}{\gamma_L}\right)}{2 + n(n-1)\gamma_L^{n-1}}$$

With the aid of Equation (B.4), we can check the relationship between competition and total welfare when an interior solution arises for information production.

Lemma B.2 (Competition and Welfare with Cross-Asset Trading)**.** *Product competition decreases total welfare* $\widetilde{W}(\widetilde{\alpha}_L, \widetilde{\alpha}_S, n)$*, i.e.,* $\frac{d\widetilde{W}(\widetilde{\alpha}_L, \widetilde{\alpha}_S, n)}{dn} < 0$*, when:*

(i) $g_S(\widetilde{\alpha}_S, n) > G_1(A_H, A_L, MC)$ *in Case 1 such that* $\widetilde{\alpha}_L = \lambda$*; and*

(ii) $g_L(\widetilde{\alpha}_L, n) > G_1(A_H, A_L, MC)$ *in Case 2 so that* $\widetilde{\alpha}_S = 0$.

We make two comments. First, Lemma B.2 verifies the validity of our key result on the non-monotonic relationship between competition and total welfare in the presence of L-traders. The numerical insights are similar and are shown in Appendix B.4.

Second, the incentive for information production can increase with the number of firms for L-traders (i.e., $\frac{d\widetilde{\alpha}_L}{dn} > 0$ for a certain range of n when $\widetilde{\alpha}_S = 0$), which differs significantly from the case for S-traders when $\lambda = 0$ (i.e., $\frac{d\widetilde{\alpha}_S}{dn} < 0$ by Proposition 2). This complexity is illustrated in Figure 10. In particular, when we move from a monopoly ($n = 1$) to a duopoly ($n = 2$), the size of the informed L-traders $\widetilde{\alpha}_L$ first increases and then decreases when n increases. To understand this non-monotonicity, we plug in $\widetilde{\alpha}_S = 0$ and use Equation (B.3) to obtain:

$$\Pi_L(\boldsymbol{\alpha}) = n\Pi_S(\alpha_L, \alpha_S) = \frac{n\widetilde{\gamma}\left(\bar{A} - MC\right)(A_H - A_L)(2\theta - 1)\left(2 + (n-1)\widetilde{\gamma}^{n-1}\right)}{2b(n+1)^2}$$

where $\widetilde{\gamma} = 1 - (2\theta - 1)\widetilde{\alpha}_L$. We can further compute:

$$\begin{aligned}\frac{\partial \Pi_L}{\partial n} &= \frac{(2\theta - 1)(A_H - A_L)\left(\bar{A} - MC\right)}{2b(n+1)^3} \\ &\quad \times \left\{\widetilde{\gamma}^n(3n-1) - 2\widetilde{\gamma}(n-1) - \left(\log \frac{1}{\widetilde{\gamma}}\right)\widetilde{\gamma}^n n(n-1)(n+1)\right\}\end{aligned}$$

Therefore, it is possible that $\frac{\partial \Pi_L}{\partial n} > 0$. For example, when α_L is sufficiently small,

$$\frac{\partial \Pi_L}{\partial n} = \frac{(2\theta - 1)(A_H - A_L)\left(\bar{A} - MC\right)}{2b(n+1)^2} + \frac{n(n-1)\widetilde{\alpha}_L}{(n+1)^2} \times O(1) > 0$$

Note that $\frac{\partial \Pi_L}{\partial n} > 0$ implies that increased competition in the product market can strengthen the incentive for L-traders to acquire and trade on private information. Intuitively, as shown in Vives (1985), the profit of firms converges to zero at a speed of $1/n$. When multiplied by the number of firms n, the trading profits for L-traders can be non-monotonicity in n. We term this the "trading opportunity effect" in cross-asset trading.

Numerical analysis. Here, we use numerical methods to verify that the basic insights still hold when there are both L-traders and S-traders in the stock market. Again, let $\Delta\widetilde{W}_n$ denote the incremental change in total welfare when the number of firms increases from $(n-1)$ to n, i.e., $\Delta\widetilde{W}_n = \widetilde{W}\left(\widetilde{\boldsymbol{\alpha}}_n, n\right) - \widetilde{W}\left(\widetilde{\boldsymbol{\alpha}}_{n-1}, n-1\right)$.

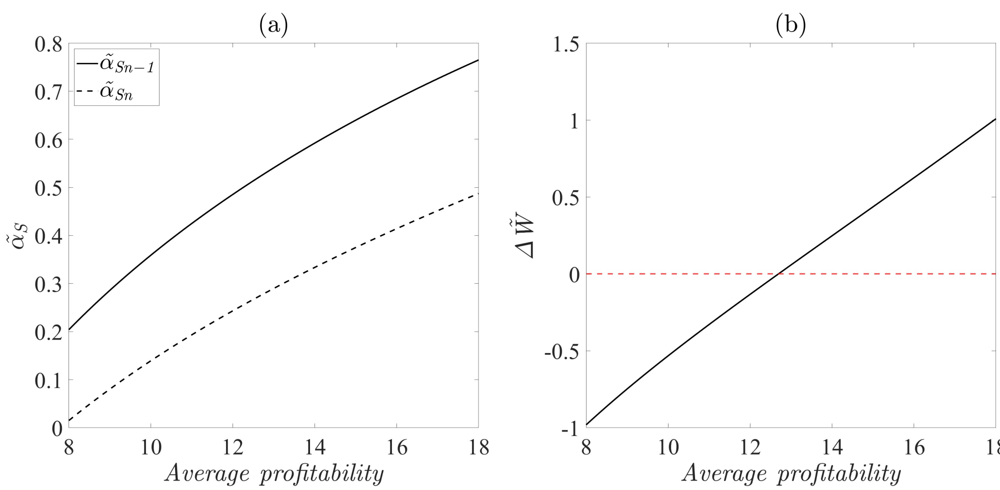


Figure 20: Average Profitability, Information Quality and Welfare.

Parameters: $A_H - A_L = 10, b = 1.5, \theta = 0.75, n = 5, MC = 3, c_L = c_S = 1.5, \lambda = 0.2$.

Remark: (Case 1) the intensity of information production for L-traders satisfies: $\widetilde{\alpha}_L = \lambda$.

First, Figure 20 illustrates how average profitability $\left(\bar{A} - MC\right)$ affects information production $\widetilde{\alpha}_S$ and total welfare $\Delta\widetilde{W}_n$ when all L-traders choose to acquire information. Specifically, similar

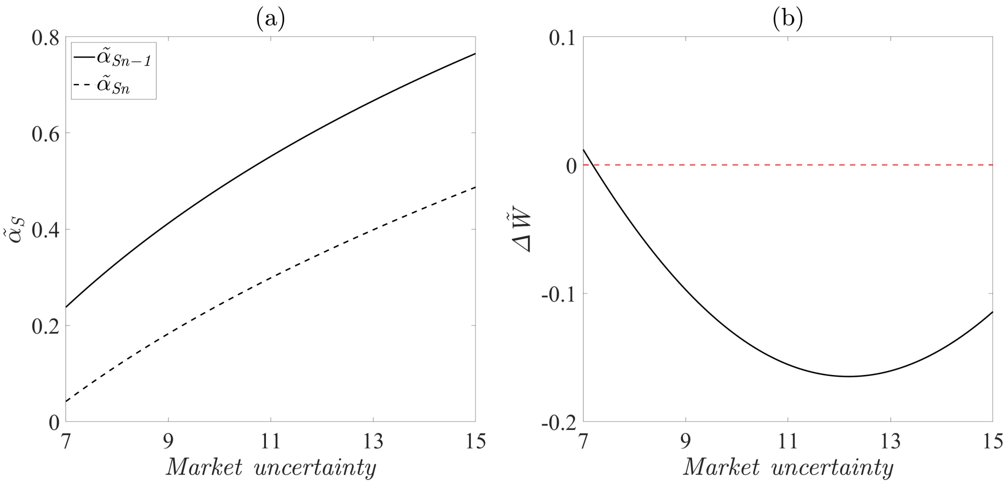


Figure 21: Uncertainty, Information Quality and Welfare.

Parameters: $\bar{A} = 15, b = 1.5, \theta = 0.75, n = 5, MC = 3, c_L = c_S = 1.5, \lambda = 0.2.$

Remark: (Case 1) the intensity of information production for L-traders satisfies: $\widetilde{\alpha}_L = \lambda$.

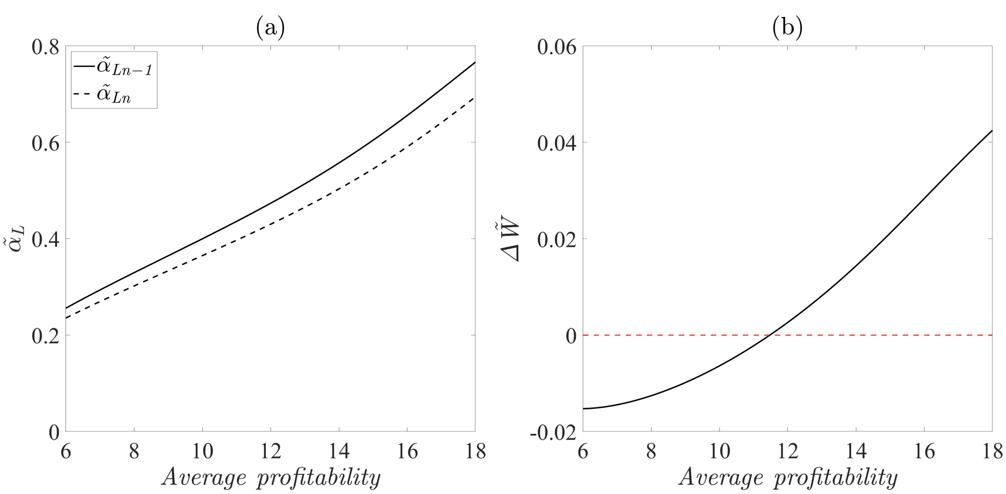


Figure 22: Average Profitability, Information Quality and Welfare.

Parameters: $A_H - A_L = 10, b = 2.5, \theta = 0.75, n = 14, MC = 6.5, c_L = c_S = 1.5, \lambda = 0.8.$

Remark: (Case 2) the intensity of information production for S-traders satisfies: $\widetilde{\alpha}_S = 0$.

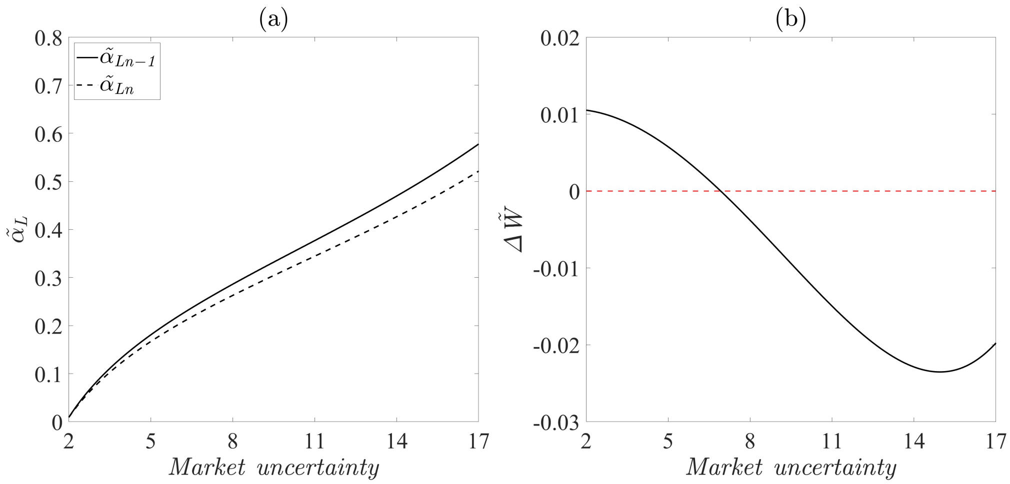


Figure 23: Uncertainty, Information Quality and Welfare.

Parameters: $A_H = 20, A_L = 10, b = 2.5, \theta = 0.75, n = 14, MC = 6.5, c_L = c_S = 1.5, \lambda = 0.8$.

Remark: (Case 2) the intensity of information production for S-traders satisfies: $\widetilde{\alpha}_S = 0$.

to Figure 7, it delivers three messages, including: (1) the intensity of information production $\widetilde{\boldsymbol{\alpha}}_n$ decreases in the number of firms n; (2) both $\widetilde{\boldsymbol{\alpha}}_n$ and $\widetilde{\boldsymbol{\alpha}}_{n-1}$ increase the average profitability $(\bar{A} - MC)$; and (3) the welfare gain $\Delta\widetilde{W}_n$ is smaller for a lower average profitability, which can even be negative when the average profitability is sufficiently low.

Furthermore, Figure 21 shows the impact of uncertainty, measured by $(A_H - A_L)$, on information production and total welfare. Specifically, it delivers three messages, including: (1) the intensity of information production $\widetilde{\boldsymbol{\alpha}}_n$ decreases in the number of firms n; (2) both $\widetilde{\boldsymbol{\alpha}}_n$ and $\widetilde{\boldsymbol{\alpha}}_{n-1}$ increase in market uncertainty $(A_H - A_L)$; and (3) the incremental welfare change can be negative when market uncertainty $(A_H - A_L)$ is high. Finally, a similar pattern ensues when all S-traders abstain from acquiring information and only a fraction of L-traders choose to produce information.

B.5 Equilibrium Analysis in Section 5.2

Equilibrium analysis. Recall that we let α_L and $\alpha_{i,S}$ denote the measure of informed L-traders and that of informed S-traders for the ith firm, and the size of L-traders is $\lambda = 0$. We first solve the equilibrium for a fixed $\boldsymbol{\alpha}$. Specifically:

$$s_i(\Omega) = \begin{cases} s_H & \text{if } \exists j : f_j \in (\gamma_j^{LS}, \infty); \\ s_M & \text{if } \forall j : f_j \in [-\gamma_j^{LS}, \gamma_j^{LS}]; \\ s_L & \text{if } \exists j : f_j \in (-\infty, -\gamma_j^{LS}). \end{cases} \tag{B.5}$$

where $s_H = \frac{(A_H - MC)^2}{(n+1)^2 b}$, $s_M = \frac{(\bar{A} - MC)^2}{(n+1)^2 b}$, $s_L = \frac{(A_L - MC)^2}{(n+1)^2 b}$, and $\gamma_i^{LS} = 1 - (2\theta - 1)(\alpha_L + \alpha_{i,S})$.

Furthermore, the ith firm optimally chooses production based on observed stock prices:

$$q_i^*(\boldsymbol{s}) = \begin{cases} q_H & \text{if } \exists j : s_j = s_H; \\ q_M & \text{if } \forall j : s_j = s_M; \\ q_L & \text{if } \exists j : s_j = s_L. \end{cases}$$

where $q_H = \frac{A_H - MC}{(n+1)b}$, $q_M = \frac{\bar{A} - MC}{(n+1)b}$ and $q_L = \frac{A_L - MC}{(n+1)b}$.

Again, for an informed L-trader k with a private signal m_k, the optimal trading strategy is to buy $y_k^j = +1$ ($y_k^j = -1$) share of each firm j when $m_k = H$ ($m_k = L$), leading to an expected trading profit given by:

$$\Pi_{L,C}(\boldsymbol{\alpha}) = \frac{n(2\theta - 1)\left(\bar{A} - MC\right)\left(A_H - A_L\right)\left(\prod_{j=1}^n \gamma_j^{LS}\right)}{2b(n+1)}$$

Similarly, for an informed S-trader k with a private signal m_k^i, the optimal trading strategy is to buy $x_k^i = +1$ shares of the ith stock when $m_k^i = H$, and sell $x_k^i = -1$ shares of the ith stock when $m_k^i = L$, leading to an expected trading profit of:

$$\Pi_{S,C}(\boldsymbol{\alpha}) = \frac{(2\theta - 1)\left(\bar{A} - MC\right)\left(A_H - A_L\right)\left(\prod_{j=1}^n \gamma_j^{LS}\right)}{2b(n+1)}$$

Here, the symbol "C" in the subscript means "cross-asset learning".

By focusing on the symmetric equilibrium (i.e., $\alpha_{i,S} = \alpha_S$), the expected profits for the L- and S-traders can be further written as: $\Pi_L(\boldsymbol{\alpha}) = n\Pi_S(\boldsymbol{\alpha})$ and

$$\Pi_{S,C}(\boldsymbol{\alpha}) = \frac{(2\theta - 1)\left(\bar{A} - MC\right)\left(A_H - A_L\right)(\gamma^{LS})^n}{2b(n+1)} \tag{B.6}$$

where $\gamma^{LS} = 1 - (2\theta - 1) \times (\alpha_L + \alpha_S)$.

Now, we turn to equilibrium information production. Define

$$\nu = \frac{1}{(2\theta - 1)} - \frac{1}{(2\theta - 1)}\left(\frac{2bc_L(n+1)}{n(2\theta - 1)\left(\bar{A} - MC\right)\left(A_H - A_L\right)}\right)^{1/n}, \quad \text{and}$$

$$\xi = \frac{1}{(2\theta - 1)} - \frac{1}{(2\theta - 1)}\left(\frac{2bc_S(n+1)}{n(2\theta - 1)\left(\bar{A} - MC\right)\left(A_H - A_L\right)}\right)^{1/n} - \lambda$$

Lemma B.3 (Information Production)**.** *The equilibrium intensity of information production* $(\widetilde{\alpha}_{L,C}, \widetilde{\alpha}_{S,C})$ *satisfies the following:*

(i) when $c_L \geq \Pi_{L,C}(0,0)$*, then* $\widetilde{\alpha}_{L,C} = \widetilde{\alpha}_{S,C} = 0$*;*

(ii) when $\Pi_{L,C}(\lambda, 0) < c_L < \Pi_{L,C}(0,0)$*, then* $\widetilde{\alpha}_{S,C} = 0$ *and* $\widetilde{\alpha}_{L,C} = \nu \in (0, \lambda)$*;*

(iii) when $c_L < \Pi_{L,C}(\lambda, 0)$ *and* $c_S \geq \Pi_{S,C}(\lambda, 0)$*, then* $\widetilde{\alpha}_{L,C} = \lambda$ *and* $\widetilde{\alpha}_{S,C} = 0$*;*

(iv) when $c_L < \Pi_{L,C}(\lambda, 0)$ *and* $\Pi_{S,C}(\lambda, 1 - \lambda) < c_S < \Pi_{S,C}(\lambda, 0)$*, then* $\widetilde{\alpha}_{L,C} = \lambda$ *and* $\widetilde{\alpha}_{S,C} = \xi \in (0, 1 - \lambda)$*; and*

(v) when $c_L < \Pi_{L,C}(\lambda, 0)$ *and* $c_S \leq \Pi_{S,C}(\lambda, 1 - \lambda)$*, then* $\widetilde{\alpha}_{L,C} = \lambda$ *and* $\widetilde{\alpha}_{S,C} = 1 - \lambda$*.*

Define $\widetilde{\boldsymbol{\alpha}}_n := \widetilde{\boldsymbol{\alpha}}(n)$. Finally, following the derivation of Equation (14), we can compute the expected total welfare $\widetilde{W}_{LS}\left(\widetilde{\boldsymbol{\alpha}}_n, n\right)$ as follows:

$$\widetilde{W}_{LS}\left(\widetilde{\boldsymbol{\alpha}}_n, n\right) = \frac{n(n+2)}{8b(n+1)^2}\left(4\left(\bar{A} - MC\right)^2 + \left(1 - (\widetilde{\gamma}^{LS})^n\right)\left(A_H - A_L\right)^2\right) \tag{B.7}$$

where $\widetilde{\gamma}^{LS} = 1 - (2\theta - 1) \times (\widetilde{\alpha}_L + \widetilde{\alpha}_S)$.

Recall that $\gamma_S = 1 - (2\theta - 1)(\lambda + \widetilde{\alpha}_S)$, $\gamma_L = 1 - \widetilde{\alpha}_L(2\theta - 1)$. Define

$$g_{S,C}(\gamma_S, n) = (\gamma_S)^n (2 + n(n+1)(n+2)).$$

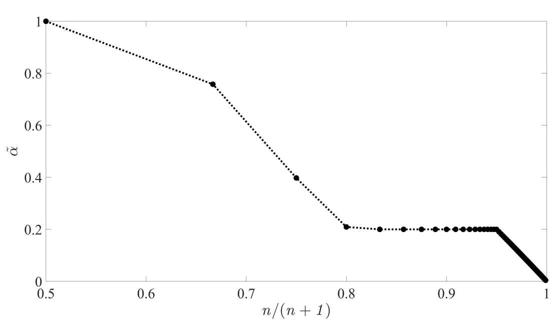
Lemma B.4 (Competition and Welfare with Cross-Asset Learning).

(i) Case 1: $\widetilde{\alpha}_{L,C} = \lambda$. Then, the total welfare decreases in the number of firms n (i.e., $\frac{d\widetilde{W}_{LS}(\widetilde{\boldsymbol{\alpha}}_n,n)}{dn} < 0$) if and only if $g_{S,C}(\gamma_S, n) > G_1(A_H, A_L, MC)$; and

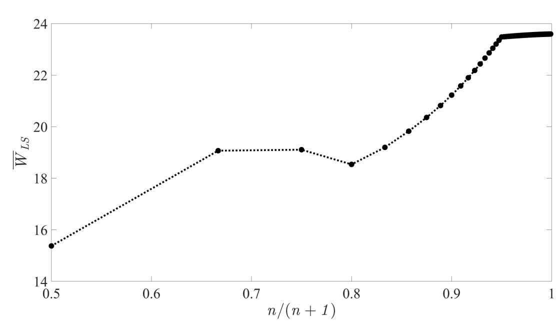
(ii) Case 2: $\widetilde{\alpha}_{S,C} = 0$. Then, the total welfare increases strictly in the number of firms n, i.e., $\frac{d\widetilde{W}_{LS}(\widetilde{\boldsymbol{\alpha}}_n,n)}{dn} > 0$.

Lemma B.4 requires several additional clarifications, given that market makers can observe the flow of orders in all stocks. First, when there are only S-traders in the stock market (i.e., $\lambda = 0$ and thus $\widetilde{\alpha}_{L,C} = 0 = \lambda$ always holds), the nonmonotonic relationship between competition and total welfare still holds. Second, the non-monotonicity also holds when the cost of information production is small such that $\widetilde{\alpha}_{L,C} = \lambda$. Note that L-traders have a stronger incentive to acquire information, compared to S-traders. Third, when there are only L-traders (i.e., $\lambda = 1$ and thus $\widetilde{\alpha}_{S,C} = 0$ always holds), the total welfare increases strictly in the number of firms n. In other words, the non-monotonic relationship between competition and total welfare holds when we allow cross-asset trading by L-traders or cross-asset learning by market makers, but not both. Intuitively, there are two economic forces behind this. On the one hand, as discussed in Section 5.1, intensified competition can improve trading profits for L-traders by granting them more trading opportunities. On the other hand, cross-asset learning provides market makers with more information, decreasing speculators' trading profits, and information production in equilibrium. In summary, both the trading opportunity effect and the cross-asset learning effect reduce the impact of the information production channel. A more detailed discussion about the divergent impact of cross-asset learning on L-traders and S-traders can be found in online Appendix B.5.

We first illustrate how competition shapes information production and total welfare when market makers can observe the order flow of all stocks.



(a) Information Production



(b) Total Welfare

Figure 24: Competition, Information Production and Total Welfare

Parameters: $\lambda = 0.2$, $\theta = 0.75$, $b = 1.5$, $A_H = 20$, $A_L = 10$, $MC = 8$, and $c_L = c_S = 1.5$.

Numerical analysis. With intensified Cournot competition ($n \uparrow$), the incentive to acquire information weakly decreases. This is illustrated in Figure 24a. First, when $n \leq 4$, an increase in n reduces the measure of informed S-traders, who have a relatively smaller incentive to acquire information. Second, when $4 < n \leq 18$, S-traders quit from acquiring information and trading on private information, while all L-traders choose to acquire information. Third, when $n \geq 18$, an increase in n further reduces the incentive for L-traders to acquire information.

Correspondingly, Figure 24b depicts total welfare when the number of firms n increases. When $n \leq 4$, total welfare first increases and then decreases and reaches a local minimum when all S-traders abstain from information production. However, when $n \geq 4$, total welfare increases strictly in the number of firms, indicating a dominant role of the market concentration channel.

Understanding the impact of cross-asset learning. By Lemma B.4, cross-asset learning affects L-traders differently from S-traders. Here, we show that this complexity is primarily caused by the combination of the trading opportunity effect and the cross-asset learning effect.

(i) Cross-asset learning effect.

Specifically, with cross-asset learning, market makers can observe the order flow of all stocks, enabling more efficient pricing against informed speculators. Thus, trading profits decrease for both L-traders and S-traders and are lower than those without cross-asset learning. Indeed, given $\widetilde{\gamma}^{LS}$ (or equivalently, $\widetilde{\alpha}_{L,C} + \widetilde{\alpha}_{S,C}$), we have:

$$\frac{\Pi_{L,C}}{\Pi_L} = \frac{\Pi_{S,C}}{\Pi_S} = f_C(n) \tag{B.8}$$

where $f_C(n) = \frac{(n+1)}{2(\widetilde{\gamma}^{LS})^{1-n}+(n-1)}$. Obviously, $f_C(n) \in (0,1)$ and $f'_C(n) < 0$. Therefore, the trading profits of an informed L-trader and an informed S-trader will shrink proportionally by a ratio of $f_C(n)$ when market makers can observe the order flow of all stocks, and this effect is more pronounced when n is large.

(ii) Trading opportunity effect.

This effect arises from the opportunity to access all stock, and thus only exists for L-traders. Unlike an S-trader with small trading opportunities, an L-trader can earn a higher trading profit by acquiring costly information, i.e., $\Pi_L = n\Pi_S$ and $\Pi_{L,C} = n\Pi_{S,C}$. Therefore, the expected trading profit of an L-trader can increase with n, especially when n is small. For example, we can verify that $\frac{\partial \Pi_L}{\partial n} > 0$ for $n = 1$, which differs from the case with an S-trader whose expected trading profit always decreases in n. However, note that $\frac{\partial \Pi_L}{\partial n} < 0$ when n is large enough. Figure 25 illustrates the pattern of trading profits with (blue dashed line) and without (red solid line) cross-asset learning by market makers.

We now examine how cross-asset learning affects the incentive for information production. We first consider S-traders, whose expected trading profits Π_S strictly decrease in n and are further reduced by cross-asset learning (i.e., $\frac{d\Pi_{S,C}}{dn} < 0$). Note that $\Pi_S = \Pi_{S,C}$ when $n = 1$ or $n \to \infty$. Then, one would expect that when n is relatively small, $\Pi_{S,C}$ decreases relatively faster than $\Pi_{S,C}$ as n increases. This is illustrated in panel (a) of Figure 25. Therefore, with cross-asset learning, the expected trading profit of an informed S-trader exhibits a higher level of sensitivity in the number of firms (n), which implies that intensified market competition can further reduce the incentive for S-traders to trade on proprietary information compared to the case without cross-asset learning.

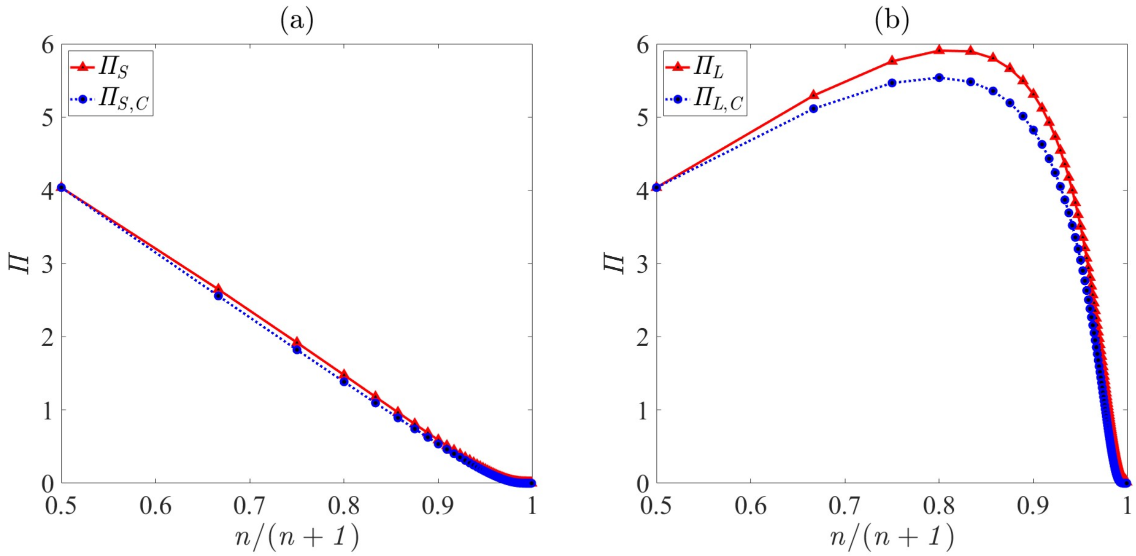


Figure 25: Trading profits with/witout cross-asset learning

Parameters: $\theta = 0.75$, $b = 2.5$, $A_H = 20$, $A_L = 10$, $MC = 6.5$, and $\widetilde{\alpha}_{L,C} + \widetilde{\alpha}_{S,C} = 0.1$.

In other words, it reinforces the informational feedback channel, leading to a stronger (negative) effect of competition on real efficiency.

Next, we consider L-traders, whose expected trading profits Π_L are non-monotonic in n. Specifically, due to the trading opportunity effect, Π_L first increases and then decreases, generating an inverted U-shape pattern when n increases. Similarly, cross-asset learning also decreases the expected trading profit $\Pi_{L,C}$ for L-traders and flattens the inverted U-shape pattern, as shown in panel (b) of Figure 25. Thus, with cross-asset learning by market makers, the expected trading profit of an informed L-trader becomes less sensitive to the number of firms (n) when n is relatively small, leading to weaker informational feedback effects. Therefore, the non-monotonic link between competition and total welfare fails because the trading opportunity effect and cross-asset learning reinforce each other.

As a final remark, Figure 25 appears to indicate that the expected trading profits Π_L and $\Pi_{L,C}$ for L-traders are relatively more sensitive to changes in n when n is large, compared to those of S-traders Π_S and $\Pi_{S,C}$. However, this does not mean that a change in n affects L-traders more than S-traders when it comes to information production. More formally, recall that $\Pi_L = n\Pi_S$ and $\Pi_{L,C} = n\Pi_{S,C}$, which further implies that: $\frac{\partial \Pi_L}{\partial \alpha_L} = n\frac{\partial \Pi_S}{\partial \alpha_S} < 0$ and $\frac{\partial \Pi_{L,C}}{\partial \alpha_L} = n\frac{\partial \Pi_{S,C}}{\partial \alpha_S} < 0$. It then follows that for L-traders, we have:

$$\frac{d\widetilde{\alpha}_L}{dn} = -\frac{1}{n} * \frac{\frac{\partial \Pi_L}{\partial n}}{\frac{\partial \Pi_S}{\partial \alpha_S}} \quad \text{and} \quad \frac{d\widetilde{\alpha}_{L,C}}{dn} = -\frac{1}{n} * \frac{\frac{\partial \Pi_{L,C}}{\partial n}}{\frac{\partial \Pi_{S,C}}{\partial \alpha_S}}$$

In contrast, for S-traders, we have:

$$\frac{d\widetilde{\alpha}_S}{dn} = -\frac{\frac{\partial \Pi_S}{\partial n}}{\frac{\partial \Pi_S}{\partial \alpha_S}} \quad \text{and} \quad \frac{d\widetilde{\alpha}_{S,C}}{dn} = -\frac{\frac{\partial \Pi_{S,C}}{\partial n}}{\frac{\partial \Pi_{S,C}}{\partial \alpha_S}}$$

Furthermore, from $\Pi_L = n\Pi_S$, we know that $\frac{\partial \Pi_L}{\partial n} = n\frac{\partial \Pi_S}{\partial n} + \Pi_S$. It follows that

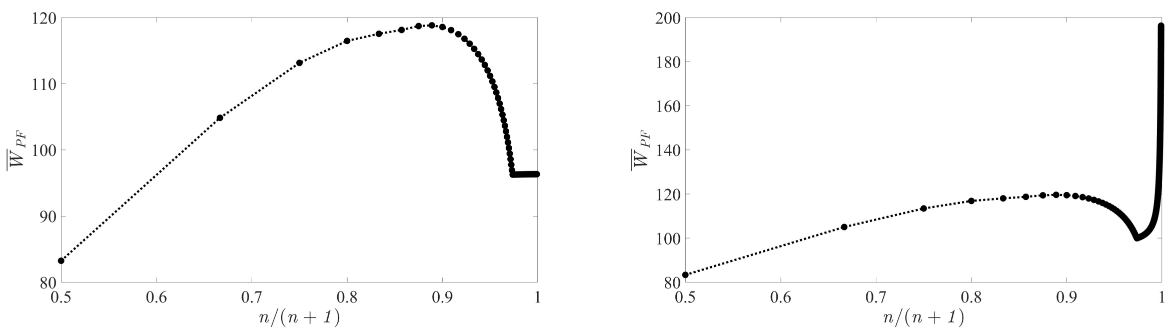
$$\frac{d\widetilde{\alpha}_L}{dn} = \frac{d\widetilde{\alpha}_S}{dn} - \frac{\Pi_S/n}{\frac{\partial \Pi_S}{\partial \alpha_L}} > \frac{d\widetilde{\alpha}_S}{dn}$$

Since $\frac{d\widetilde{\alpha}_S}{dn} < 0$, we have $\left|\frac{d\widetilde{\alpha}_L}{dn}\right| < \left|\frac{d\widetilde{\alpha}_S}{dn}\right|$, when $\frac{d\widetilde{\alpha}_L}{dn} < 0$. Similarly, with cross-asset learning, we also have: $\left|\frac{d\widetilde{\alpha}_{L,C}}{dn}\right| < \left|\frac{d\widetilde{\alpha}_{S,C}}{dn}\right|$, when $\frac{d\widetilde{\alpha}_{L,C}}{dn} < 0$. Thus, intensified market competition will negatively affect S-traders more than L-traders in terms of information production.

B.6 Formal Analysis for Section 5.3

This section provides a formal analysis for Section 5.3. Specifically, we first present a non-monotonic welfare result and then depict the relationship between competition and total welfare when investor welfare is included. Recall that $\Phi(m)$ is defined in Proposition 3, and define $m_0 = \inf\{m \in \mathbb{N} : \Phi(m) \geq 1\}$. Define $\tilde{c} = \frac{2bc}{(\bar{A}-MC)^2}$.

Lemma B.5 (Informational Feedback & Over-Competition)**.** *Assume $B(n) = B_0$ for some constant B_0. Suppose that $\Phi(m) - m * \tilde{c} - m > 0$ for some $m \geq m_0$. Then, for any $n \geq N(m) > m$, $\overline{W}(\widehat{\alpha}_m, m) > \overline{W}(\widehat{\alpha}_n, n)$ holds for any $c \in [\bar{c}_n, \underline{c}_m)$ with $\bar{c}_n < \underline{c}_m$.*



(a) $B(n) = 1$

(b) $B(n) = 0.1 * n$

Figure 26: Competition & Total Welfare (with Investor Welfare)

Parameters: $\theta = 0.75$, $b = 1.5$, $A_H = 30$, $A_L = 10$, $MC = 3$, and $c = 1.5$.

Figure 26 illustrates the relationship between product competition and total welfare when investor welfare is included in the calculation. Specifically, when the aggregate benefit of liquidity trading is fixed, Figure 26a demonstrates a non-monotonic pattern between competition and total welfare, which is similar to Figure 4. In particular, total welfare first increases and then decreases, and is maximized at $n = 8$. Similarly, Figure 26b illustrates the relationship by specifying the aggregate benefit of liquidity trading as an increasing function of the number of stocks, i.e., $B(n) = 0.1 * n$. The total welfare is also non-monotonic and becomes infinitely large due to the unbounded return from liquidity trading.

B.7 Skipped Proofs in the Online Appendix

B.7.1 Proof of Lemma B.1

Proof. We first state two properties: (a) We compute the following derivatives, including:

$$\frac{\partial \Pi_L(\alpha_L, \alpha_S)}{\partial \alpha_L} = -\frac{n(A_H - A_L)\left(\bar{A} - MC\right)(2\theta - 1)^2\left(2 + n(n-1)(\gamma^{LS})^{n-1}\right)}{2b(n+1)^2} < 0;$$
$$\frac{\partial \Pi_S(\alpha_L, \alpha_S)}{\partial \alpha_S} = -\frac{(A_H - A_L)\left(\bar{A} - MC\right)(2\theta - 1)^2\left(2 + n(n-1)(\gamma^{LS})^{n-1}\right)}{2b(n+1)^2} < 0.$$

and (b) Note that $\Pi_L(\alpha_L, \alpha_S) = n\Pi_S(\alpha_L, \alpha_S)$.

Now, we prove the lemma. First, consider $c_L \geq \Pi_L(0,0)$. Obviously, $\widetilde{\alpha}_L = 0$. Meanwhile, since $c_S \geq c_L$ and $\Pi_L(0,0) \geq \Pi_S(0,0)$, $\widetilde{\alpha}_S = 0$.

Second, consider $\Pi_L(\lambda, 0) < c_L < \Pi_L(0,0)$. By the derivative $\frac{\partial \Pi_L(\alpha_L, \alpha_S)}{\partial \alpha_L} < 0$ and continuity, there exists a unique $\widetilde{\alpha}_L$ such that $\Pi_L(\widetilde{\alpha}_L, 0) = c_L$. Furthermore, given $\widetilde{\alpha}_L$, $\frac{\partial \Pi_S(\alpha_L, \alpha_S)}{\partial \alpha_S} < 0$ implies that $\Pi_S(\widetilde{\alpha}_L, 0) > \Pi_S(\widetilde{\alpha}_L, \alpha_S)$ for any $\alpha_S > 0$. Thus, $c_S \geq c_L = \Pi_L(\widetilde{\alpha}_L, 0) \geq \Pi_S > \Pi_S(\widetilde{\alpha}_L, \alpha_S)$ for any $\alpha_S > 0$. Therefore, $\widetilde{\alpha}_S = 0$.

Third, consider $c_L < \Pi_L(\lambda, 0)$ and $c_S \geq \Pi_S(\lambda, 0)$. Obviously, $(\widetilde{\alpha}_L, \widetilde{\alpha}_S) = (\lambda, 0)$. Furthermore, this is also the unique equilibrium. If not, consider any equilibrium $(\widetilde{\alpha}_L, \widetilde{\alpha}_S)$ with $\widetilde{\alpha}_S > 0$. Note that by property (b), we can infer: $\Pi_L(\widetilde{\alpha}_L, \widetilde{\alpha}_S) > \Pi_S(\widetilde{\alpha}_L, \widetilde{\alpha}_S) \geq c_S \geq c_L$, which implies that $\widetilde{\alpha}_L = \lambda$, which in turn implies that $\widetilde{\alpha}_S = 0$.

Fourth, consider $c_L < \Pi_L(\lambda, 0)$ and $\Pi_S(\lambda, 1-\lambda) < c_S < \Pi_S(\lambda, 0)$. We have shown above that if $\widetilde{\alpha}_S > 0$, then $\widetilde{\alpha}_L = \lambda$. Given that $c_L < \Pi_L(\lambda, 0)$, we can infer that $\widetilde{\alpha}_L = \lambda$. Given this and the assumed condition $\Pi_S(\lambda, 1-\lambda) < c_S < \Pi_S(\lambda, 0)$, by the monotonicity and continuity of $\Pi_S(\alpha_L, \alpha_S)$, there is a unique $\widetilde{\alpha}_S \in (0, 1-\lambda)$ such that $\Pi_S(\lambda, \widetilde{\alpha}_S) = c_S$.

Fifth, consider $c_L < \Pi_L(\lambda, 0)$ and $c_S \leq \Pi_S(\lambda, 1-\lambda)$. Obviously, by the facts $c_S \geq c_L$ and $\Pi_L \geq \Pi_S$, we have: $\widetilde{\alpha}_L = \lambda$ and $\widetilde{\alpha}_S = 1 - \lambda$. The proof concludes. □

B.7.2 Proof of Lemma B.2

Proof. **Case 1: $\widetilde{\alpha}_L = \lambda$.** We can rewrite $\widetilde{W}(\widetilde{\alpha}_L, \widetilde{\alpha}_S, n)$ and $\Pi_S(\widetilde{\alpha}_L, \widetilde{\alpha}_S)$ as:

$$\widetilde{W}(\widetilde{\alpha}_S, n) = \frac{n(n+2)}{8b(n+1)^2}\left(4(\bar{A} - MC)^2 + (1 - {\gamma_S}^n)(A_H - A_L)^2\right),$$
$$\Pi_S(\widetilde{\alpha}_S, n) = \frac{\gamma_S(2\theta - 1)(A_H - A_L)(\bar{A} - MC)\left(2 + (\gamma_S)^{n-1}(n-1)\right)}{2b(n+1)^2}$$

where $\gamma_S = 1 - (\lambda + \widetilde{\alpha}_S)(2\theta - 1)$.

Then, we can calculate the following partial derivatives:

$$\frac{\partial \widetilde{W}}{\partial \widetilde{\alpha}_S} = \frac{n^2(n+2)\gamma_S^{n-1}(2\theta-1)(A_H-A_L)^2}{8b(n+1)^2},$$
$$\frac{\partial \widetilde{W}}{\partial n} = \frac{n(n+2)\gamma_S^n(A_H-A_L)^2\ln(1/\gamma_S)}{8b(n+1)^2} + \frac{2\left((A_H-MC)^2+(A_L-MC)^2\right)-\gamma_S^n(A_H-A_L)^2}{4b(n+1)^3}$$
$$\frac{\partial \Pi_S}{\partial \widetilde{\alpha}_S} = -\frac{(2\theta-1)^2\left((A_H-MC)^2-(A_L-MC)^2\right)\left(2+n(n-1){\gamma_S}^{n-1}\right)}{4b(n+1)^2}$$
$$\frac{\partial \Pi_S}{\partial n} = -\frac{(2\theta-1)\left((A_H-MC)^2-(A_L-MC)^2\right)\left(4\gamma_S+\gamma_S^n\left(n-3-(n^2-1)\ln\gamma_S\right)\right)}{4b(n+1)^3}$$

By the implicit function theorem, we have:

$$\frac{\partial \widetilde{\alpha}_S}{\partial n} = -\frac{\partial \Pi_S/\partial n}{\partial \Pi_S/\widetilde{\alpha}_S} = -\frac{\gamma_S^n \times \left(\left(4\gamma_S^{1-n}+(n-3)\right)/(n+1)+(n-1)\ln(1/\gamma_S)\right)}{(2\theta-1)\left(2+n(n-1)\gamma_S^{n-1}\right)}$$

which further implies:

$$\frac{d\widetilde{W}\left(\widetilde{\alpha}_{S,C},n\right)}{dn} = \frac{\partial \widetilde{W}}{\partial n} + \frac{\partial \widetilde{W}}{\partial \widetilde{\alpha}_S}\frac{\partial \widetilde{\alpha}_S}{\partial n} = \frac{(A_H-A_L)^2\left(G_1-g_S\left(\widetilde{\alpha}_S,n\right)\right)}{8b(n+1)^3},$$

Thus, $\frac{d\widetilde{W}\left(\widetilde{\alpha}_{S,C},n\right)}{dn} < 0$ if and only if $g_S\left(\widetilde{\alpha}_S,n\right) > G_1$.

Case 2: $\widetilde{\alpha}_S = 0$. We can rewrite $\widetilde{W}(\widetilde{\alpha}_L,\widetilde{\alpha}_S,n)$ and $\Pi_L(\widetilde{\alpha}_L,\widetilde{\alpha}_S)$ as:

$$\widetilde{W}(\widetilde{\alpha}_L,n) = \frac{n(n+2)}{8b(n+1)^2}\left(4(\bar{A}-MC)^2+(1-(\gamma_L)^n)(A_H-A_L)^2\right),$$
$$\Pi_S(\widetilde{\alpha}_L,n) = \frac{\gamma_S(2\theta-1)(A_H-A_L)(\bar{A}-MC)\left(2+(\gamma_L)^{n-1}(n-1)\right)}{2b(n+1)^2}$$

where $\gamma_L = 1-\widetilde{\alpha}_L\times(2\theta-1)$.

Then, we can calculate the following partial derivatives:

$$\frac{\partial \widetilde{W}}{\partial \widetilde{\alpha}_L} = \frac{n^2(n+2)\gamma_L^{n-1}(2\theta-1)(A_H-A_L)^2}{8b(n+1)^2},$$
$$\frac{\partial \widetilde{W}}{\partial n} = \frac{n(n+2)\gamma_L^n(A_H-A_L)^2\ln(1/\gamma_L)}{8b(n+1)^2} + \frac{2\left((A_H-MC)^2+(A_L-MC)^2\right)-\gamma_L^n(A_H-A_L)^2}{4b(n+1)^3}$$

$$\frac{\partial \Pi_L}{\partial \widetilde{\alpha}_L} = -\frac{n(2\theta-1)^2\left((A_H-MC)^2-(A_L-MC)^2\right)\left(2+n(n-1){\gamma_L}^{n-1}\right)}{4b(n+1)^2}$$
$$\frac{\partial \Pi_L}{\partial n} = -\frac{(2\theta-1)\left((A_H-MC)^2-(A_L-MC)^2\right)\left(2(1-n)\gamma_L+\gamma_L^n\left((3n-1)+n(n^2-1)\ln\gamma_L\right)\right)}{4b(n+1)^3}$$

By the implicit function theorem, we have:

$$\frac{\partial \widetilde{\alpha}_L}{\partial n} = -\frac{\partial \Pi_L/\partial n}{\partial \Pi_L/\widetilde{\alpha}_L} = \frac{2\gamma_L\times(1-n)+(\gamma_L)^n\left((3n-1)-n(n^2-1)\ln(1/\gamma_L)\right)}{n(n+1)(2\theta-1)\left(2+n(n-1)\gamma_L^{n-1}\right)}$$

which further implies:

$$\frac{d\widetilde{W}\left(\widetilde{\alpha}_L, n\right)}{dn}=\frac{\partial \widetilde{W}}{\partial n}+\frac{\partial \widetilde{W}}{\partial \widetilde{\alpha}_L} \frac{\partial \widetilde{\alpha}_L}{\partial n}=\frac{n\left(A_H-A_L\right)^2\left(G_1-g_L\left(\widetilde{\alpha}_L, n\right)\right)}{8 b n(n+1)^3},$$

Thus, $\frac{d\widetilde{W}(\widetilde{\alpha}_L,n)}{dn}<0$ if and only if $g_L\left(\widetilde{\alpha}_L, n\right)>G_1$. The proof concludes. □

B.7.3 Proof of Lemma B.3

Proof. We first state two important properties: (a) $\Pi_{L,C}\left(\alpha_L, \alpha_S\right)=n \Pi_{S,C}\left(\alpha_L, \alpha_S\right)$; and (b) we compute the following derivatives, including $\frac{\partial \Pi_{L,C}(\alpha_L,\alpha_S)}{\partial \alpha_{L,C}}$ and $\frac{\partial \Pi_{S,C}(\alpha_L,\alpha_S)}{\partial \alpha_{S,C}}$. Based on the expressions for trading profits of an informed L-trader and an informed S-trader, we have:

$$\begin{aligned}
\frac{\partial \Pi_{L,C}\left(\alpha_L, \alpha_S\right)}{\partial \alpha_{L,C}} &=-\frac{n^2\left(\gamma^{LS}\right)^{n-1}(2\theta-1)^2\left(\bar{A}-MC\right)\left(A_H-A_L\right)}{2(n+1) b}<0 \\
\frac{\partial \Pi_{S,C}\left(\alpha_L, \alpha_S\right)}{\partial \alpha_{S,C}} &=-\frac{n\left(\gamma^{LS}\right)^{n-1}(2\theta-1)^2\left(\bar{A}-MC\right)\left(A_H-A_L\right)}{2(n+1) b}<0
\end{aligned}$$

Next, we prove the lemma. First, consider $c_L \geq \Pi_{L,C}(0,0)$. Obviously, $\widetilde{\alpha}_{L,C}=0$. Meanwhile, since $c_S \geq c_L$ and $\Pi_{L,C}(0,0)=n\Pi_{S,C}(0,0)$, we can deduce that $\widetilde{\alpha}_{S,C}=0$.

Second, consider $\Pi_{L,C}(\lambda,0)<c_L<\Pi_{L,C}(0,0)$. By the derivative $\frac{\partial \Pi_{L,C}(\alpha_L,\alpha_S)}{\partial \alpha_L}<0$, and continuity, there exists a unique $\widetilde{\alpha}_{L,C}$ such that $\Pi_{L,C}\left(\widetilde{\alpha}_{L,C}, 0\right)=c_L$. By solving the equation $\Pi_{L,C}\left(\widetilde{\alpha}_{L,C}, 0\right)=c_L$, we have $\widetilde{\alpha}_{L,C}=\nu$. Furthermore, given $\widetilde{\alpha}_{L,C}$, $\frac{\partial \Pi_{S,C}(\alpha_L,\alpha_S)}{\partial \alpha_S}<0$ implies that $\Pi_{S,C}\left(\widetilde{\alpha}_{L,C}, 0\right)>\Pi_{S,C}\left(\widetilde{\alpha}_{L,C}, \alpha_S\right)$ for any $\alpha_S>0$. Thus, $c_S \geq c_L=\Pi_{L,C}\left(\widetilde{\alpha}_{L,C}, 0\right)>\Pi_{S,C}\left(\widetilde{\alpha}_{L,C}, \alpha_S\right)$ for any $\alpha_S>0$. Therefore, $\widetilde{\alpha}_{S,C}=0$.

Third, consider $c_L \leq \Pi_{L,C}(\lambda,0)$ and $c_S \geq \Pi_{S,C}(\lambda,0)$. Obviously, $\left(\widetilde{\alpha}_{L,C}, \widetilde{\alpha}_{S,C}\right)=(\lambda,0)$. Furthermore, this is also the unique equilibrium. If not, consider any equilibrium $\left(\widetilde{\alpha}_{L,C}, \widetilde{\alpha}_{S,C}\right)$ with $\widetilde{\alpha}_{S,C}>0$. Note that by property (b), we can infer: $\Pi_{L,C}\left(\widetilde{\alpha}_{L,C}, \widetilde{\alpha}_{S,C}\right)>\Pi_{S,C}\left(\widetilde{\alpha}_{L,C}, \widetilde{\alpha}_{S,C}\right) \geq c_S \geq c_L$, which implies that $\widetilde{\alpha}_{L,C}=\lambda$, which in turn implies that $\widetilde{\alpha}_{S,C}=0$.

Fourth, consider $c_L \leq \Pi_{L,C}(\lambda,0)$ and $\Pi_{S,C}(\lambda,1-\lambda)<c_S<\Pi_{S,C}(\lambda,0)$. We have shown above that if $\widetilde{\alpha}_{S,C}>0$, then $\widetilde{\alpha}_{L,C}=\lambda$. Given that $c_L \leq \Pi_{L,C}(\lambda,0)$, we can infer that $\widetilde{\alpha}_{L,C}=\lambda$. Given this and the assumed condition $\Pi_{S,C}(\lambda,1-\lambda)<c_S<\Pi_{S,C}(\lambda,0)$, by the monotonicity and continuity of $\Pi_{S,C}\left(\widetilde{\alpha}_{L,C}, \widetilde{\alpha}_{S,C}\right)$, there is a unique $\widetilde{\alpha}_{S,C} \in(0,1-\lambda)$ such that $\Pi_{S,C}\left(\lambda, \widetilde{\alpha}_{S,C}\right)=c_S$. By solving $\Pi_{S,C}\left(\lambda, \widetilde{\alpha}_{S,C}\right)=c_S$, we have $\widetilde{\alpha}_{S,C}=\xi$.

Fifth, consider $c_L \leq \Pi_{L,C}(\lambda,0)$ and $c_S \leq \Pi_{S,C}(\lambda,1-\lambda)$. Obviously, by the facts $c_S \geq c_L$ and $\Pi_{L,C}>\Pi_{S,C}$, we have: $\widetilde{\alpha}_{L,C}=\lambda$ and $\widetilde{\alpha}_{S,C}=1-\lambda$. The proof concludes. □

B.7.4 Proof of Lemma B.4

Proof. We first state two important properties: (a) $\Pi_{L,C}\left(\alpha_L, \alpha_S\right)=n \Pi_{S,C}\left(\alpha_L, \alpha_S\right)$; and (b) we compute the following derivatives, including $\frac{\partial \Pi_{L,C}(\alpha_L,\alpha_S)}{\partial \alpha_{L,C}}$ and $\frac{\partial \Pi_{S,C}(\alpha_L,\alpha_S)}{\partial \alpha_{S,C}}$. Based on the expressions for trading profits of an informed L-trader and an informed S-trader, we have:

$$\frac{\partial \Pi_{L,C}\left(\alpha_L, \alpha_S\right)}{\partial \alpha_{L,C}} = -\frac{n^2\left(\gamma^{LS}\right)^{n-1}(2\theta-1)^2\left(\bar{A}-MC\right)\left(A_H-A_L\right)}{2(n+1)b} < 0$$
$$\frac{\partial \Pi_{S,C}\left(\alpha_L, \alpha_S\right)}{\partial \alpha_{S,C}} = -\frac{n\left(\gamma^{LS}\right)^{n-1}(2\theta-1)^2\left(\bar{A}-MC\right)\left(A_H-A_L\right)}{2(n+1)b} < 0$$

Now, we prove the lemma.

Case 1: $\widetilde{\alpha}_{L,C} = \lambda$**.** We can rewrite $\widetilde{W}_{LS}\left(\widetilde{\boldsymbol{\alpha}}_n, n\right)$ and $\Pi_{L,C}(\boldsymbol{\alpha}_n)$ as:

$$\widetilde{W}_{LS}(\widetilde{\alpha}_S, n) = \frac{n(n+2)}{8b(n+1)^2}\left(4(\bar{A}-MC)^2 + (1-\gamma_S^n)(A_H-A_L)^2\right),$$
$$\Pi_{S,C}(\widetilde{\alpha}_S, n) = \frac{\gamma_S^n(2\theta-1)(A_H-A_L)(\bar{A}-MC)}{2b(n+1)}$$

where $\gamma_S = 1-(\lambda+\widetilde{\alpha}_S)(2\theta-1)$.

Then, we can calculate the following partial derivatives:

$$\frac{\partial \widetilde{W}_{LS}}{\partial \widetilde{\alpha}_{S,C}} = \frac{\gamma_S^{n-1} n^2(n+2)(2\theta-1)(A_H-A_L)^2}{8b(n+1)^2},$$
$$\frac{\partial \widetilde{W}_{LS}}{\partial n} = \frac{\gamma_S^n n(n+2)(A_H-A_L)^2\ln(1/\gamma_S)}{8b(n+1)^2} + \frac{2\left((A_H-MC)^2+(A_L-MC)^2\right)-\gamma_S^n(A_H-A_L)^2}{4b(n+1)^3}$$
$$\frac{\partial \Pi_{S,C}}{\partial \widetilde{\alpha}_{S,C}} = -\frac{n\gamma_S^{n-1}\left(\bar{A}-MC\right)(A_H-A_L)(2\theta-1)^2}{2b(n+1)}$$
$$\frac{\partial \Pi_{S,C}}{\partial n} = -\frac{\gamma_S^n(2\theta-1)\left(\bar{A}-MC\right)\left(A_H-A_L\right)\left(1+(n+1)\left(\ln 1/\gamma_S\right)\right)}{2b(n+1)^2}$$

By the implicit function theorem, we have:

$$\frac{\partial \widetilde{\alpha}_{S,C}}{\partial n} = -\frac{\partial \Pi_{S,C}/\partial n}{\partial \Pi_{S,C}/\widetilde{\alpha}_{S,C}} = -\frac{\gamma_S\left(1+\ln(1/\gamma_S)\right)}{n(2\theta-1)}$$

which further implies:

$$\frac{d\widetilde{W}_{LS}\left(\widetilde{\alpha}_{S,C}, n\right)}{dn} = \frac{\partial \widetilde{W}_{LS}}{\partial n} + \frac{\partial \widetilde{W}_{LS}}{\partial \widetilde{\alpha}_{S,C}}\frac{\partial \widetilde{\alpha}_{S,C}}{\partial n} = \frac{\left(A_H-A_L\right)^2\left(G_1 - g_{S,C}\left(\gamma_S, n\right)\right)}{8b(n+1)^3}.$$

Thus, $\frac{d\widetilde{W}_{LS}\left(\widetilde{\alpha}_{S,C}, n\right)}{dn} < 0$ if and only if $g_{S,C}(\gamma_S, n) > G_1$.

Case 2: $\widetilde{\alpha}_{S,C} = 0$**.** We can rewrite $\widetilde{W}_{LS}\left(\widetilde{\boldsymbol{\alpha}}_n, n\right)$ and $\Pi_{L,C}(\boldsymbol{\alpha}_n)$ as:

$$\widetilde{W}_{LS}(\widetilde{\alpha}_{L,C}, n) = \frac{n(n+2)}{8b(n+1)^2}\left(4(\bar{A}-MC)^2 + (1-\gamma_L^n)(A_H-A_L)^2\right),$$
$$\Pi_{S,C}(\widetilde{\alpha}_{L,C}, n) = \frac{n\gamma_L^n(2\theta-1)\left(\bar{A}-MC\right)\left(A_H-A_L\right)}{2b(n+1)}$$

where $\gamma_L = 1-\widetilde{\alpha}_L \times (2\theta-1)$.

Then, we can calculate the following partial derivatives:

$$
\begin{aligned}
\frac{\partial \widetilde{W}_{LS}}{\partial \widetilde{\alpha}_L} &= \frac{\gamma_L^{n-1} n^2 (n+2)(2\theta-1)(A_H - A_L)^2}{8b(n+1)^2}, \\
\frac{\partial \widetilde{W}_{LS}}{\partial n} &= \frac{n(n+2)\gamma_L^n (A_H - A_L)^2 \ln(1/\gamma_L)}{8b(n+1)^2} + \frac{2\left((A_H - MC)^2 + (A_L - MC)^2\right) - \gamma_L^n (A_H - A_L)^2}{4b(n+1)^3} \\
\frac{\partial \Pi_{L,C}}{\partial \widetilde{\alpha}_L} &= -\frac{n^2 \gamma_L^{n-1}\left(\bar{A} - MC\right)(A_H - A_L)(2\theta-1)^2}{2b(n+1)} \\
\frac{\partial \Pi_{L,C}}{\partial n} &= \frac{\gamma_L^n (2\theta-1)\left(\bar{A} - MC\right)(A_H - A_L)\left(1 - n(n+1)ln\left(1/\gamma_L\right)\right)}{2b(n+1)^2}
\end{aligned}
$$

By the implicit function theorem, we have:

$$
\frac{\partial \widetilde{\alpha}_{L,C}}{\partial n} = -\frac{\partial \Pi_{L,C}/\partial n}{\partial \Pi_{L,C}/\partial \widetilde{\alpha}_{L,C}} = \frac{\gamma_L \left(1 - n(n+1)\ln(1/\gamma_L)\right)}{n^2(n+1)(2\theta-1)}
$$

which further implies:

$$
\frac{d\widetilde{W}_{LS}\left(\widetilde{\alpha}_{L,C}, n\right)}{dn} = \frac{\partial \widetilde{W}_{LS}}{\partial n} + \frac{\partial \widetilde{W}_{LS}}{\partial \widetilde{\alpha}_{L,C}} \frac{\partial \widetilde{\alpha}_{L,C}}{\partial n} = \frac{4\left((A_H - MC)^2 + (A_L - MC)^2\right) + n\gamma_L^n (A_H - A_L)^2}{8b(n+1)^3}
$$

Obviously, $\frac{d\widetilde{W}_{LS}\left(\widetilde{\alpha}_{L,C}, n\right)}{dn} > 0$. The proof concludes. □

B.7.5 Proof of Lemma B.5

Proof. First, note that $B(n) = B_0$ eliminates the impact of the benefits of liquidity trading and thus we can focus on the information cost. Second, $\Phi(m) - m * \tilde{c} > 0$ holds for some $m \geq m_0$ for $\frac{2b}{(\bar{A}-MC)^2}$ sufficiently small since $\Phi(m) > 1$ for $m \geq m_0 + 1$. Third, note that

$$
\frac{\overline{W}\left(\widehat{\alpha}_m, m\right)}{\overline{W}\left(\widehat{\alpha}_n, n\right)} = \frac{\left(1 - \frac{1}{(m+1)^2}\right) * \left(1 + \mu * \left(1 - (2-2\theta)^m\right)\right) - m * \tilde{c}}{\left(1 - \frac{1}{(n+1)^2}\right)}
$$

Then, the remaining proof follows from that of Proposition 3. The proof concludes. □